Constructional Steelwork Simply Explained

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PREFACE

THERE are many for whom this little book is too elementary, but I hope there are some to whom it will be useful. I have already had some very charming letters from readers of *The Builder*, in whose pages these articles first appeared, which make me think this may be so—in which case the time taken in writing it is amply repaid.

I hope it may be specially useful to my students at the A.A. School and to all students, draughtsmen, and assistants interested in the design of steelwork in a practical manner.

It is difficult to know just where best to begin and where to end—and if all my readers do not find I have chosen my limits just where they would have them I hope they will realize the difficulty of pleasing everybody, and be indulgent.

I have used no mathematics. For this I feel fairly confident I shall be forgiven—and for those who cannot dissociate engineering from ' maths '—well, most text-books should satisfy them.

In this I hope I do not appear to be wanting in respect to mathematics, which is, of course, a tool of inestimable value in engineering. Obviously any work involving new problems is greatly facilitated by having this tool both sharp and handy. Nevertheless, the really basic things which need to be understood first are explainable without them. I have found in practice more errors of design result from failure to understand the simple things, and stupid mistakes in simple arithmetic, than from any other cause.

Just one word to the student. Don't work out stresses to

many significant figures. It gives you a sense of accuracy which is purely fictitious and misleading. If you know a stress within ten per cent. you are doing very well, and probably well within your need. The strength of practical materials varies by this amount. And can you guarantee the accuracy of all your formulae, and the assumptions on which they were based, finer than this ? And what allowance have you made for residual cooling stresses, unequal strain in rolling, stresses due to temperature changes, shrinkages, wind, &c., &c. ?

Let us be practical and preserve a sense of proportion. If you want great accuracy you must certainly estimate the effect of these matters and many others. But let us get the simple things clear first.

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CHAPTER I

ELEMENTARY PROPERTIES OF STEEL

THE application of steel to constructional work clearly involves a knowledge of the properties of the former as well as the requirements and conditions of the latter. It will perhaps be well to concentrate on the first of these to begin with.

Let us, firstly, make an experiment (which, incidentally, is a practical one, repeated thousands of times a day in the routine testing of steel) which brings out many of the important properties.

We take a piece of commercial mild steel, say, 1 in. in diameter and about 16 in. long. We grip the two ends in the jaws of a testing machine, so arranged that the more we pull on the rod the tighter the wedges grip against the rod. The exact arrangement of the grip need not concern us here too intimately, but the general idea is set out diagrammatically in Fig. 1. The wedges are made of hardened steel, serrated on the inside so as to grip the rod better, and slightly curved vertically on the back so as to rock into a position which will grip the rod evenly for the whole length of the wedges.

The pull is transmitted from the testing machine to the grip heads by a hardened steel knife edge bearing on a hardened steel plate, as also is the beam of the testing machine. This not only ensures a direct pull on the specimen without bending (as would exist if there were friction at the joint), but also enables the distance from the line of pull in the test rod to the fulcrum about which the beam swings to be capable of accurate measurement.

It is remarkable that these knife edges will stand a load of 5 tons per lin. in., which, having regard to the extremely small area of contact, involves a stress of hundreds of tons per sq. in. Naturally, only very hard steel would stand this without indentation.

The general arrangement of a testing machine is set out diagrammatically in Fig. 2. It consists essentially of two parts. The first is a mechanism for applying load to the bottom end of the specimen. This is generally done either by an electric motor acting through suitable reduction gearing, so that

rotation of the motor can produce a very large pull (may be 100 tons, depending on the limit of the particular machine) with, of course, a correspondingly small movement on the lower end of the specimen. Other means, such as hydraulic, or belt drive and gearing, are also sometimes used.

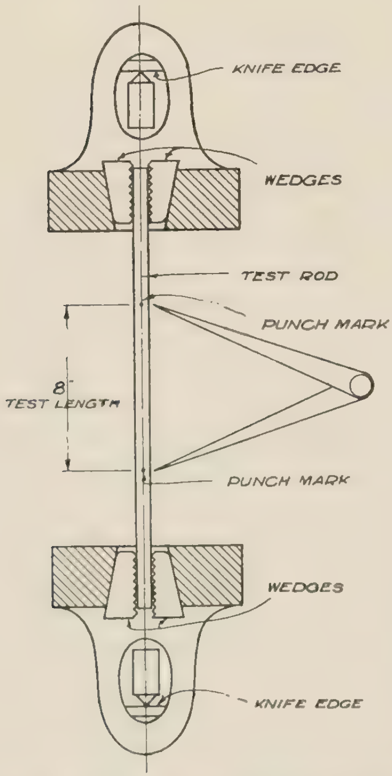


FIG. 1. Grip of Rod in Testing Machine.

The second component is the means whereby the applied load is accurately measured. This generally consists of a large beam, resting on the fulcrum already referred to, with the pull from the test specimen a few inches on one side, and a jockey weight on the other. This jockey weight can be moved along the beam until it floats (so that the end moves to a central position between the top and bottom stops). The position of the jockey weight, when balance has been obtained, measures the load on the specimen which may, therefore, be read off an horizontal scale fixed on the beam.

The jockey weight generally runs on rollers and has its movement controlled by a screw, actuated by a wheel at a convenient height through suitable bevel gearing.

On a hundred ton testing machine the distance between the knife edges may be 4 in., in which case a load of 2 tons at the end of a beam (200 in. from the fulcrum) will produce balance, since

$$4 \text{ in.} \times 100 \text{ tons} = 200 \text{ in.} \times 2 \text{ tons}.$$

This is a simple application of the principle of simple levers, according to which (and a simple test will confirm it) the load multiplied by its distance on one side of the fulcrum is equal to the load multiplied by its distance on the other, when balance

is obtained. In practice the beam itself is no light weight and has to be allowed for when fixing the scale.

The actual test proceeds as follows: The jockey weight is run along to a position near the fulcrum. The specimen is fixed in the grips, after the perfect balance of the beam has been checked (and adjusted if need be) when the jockey weight is on the zero position. A small load is then applied to the

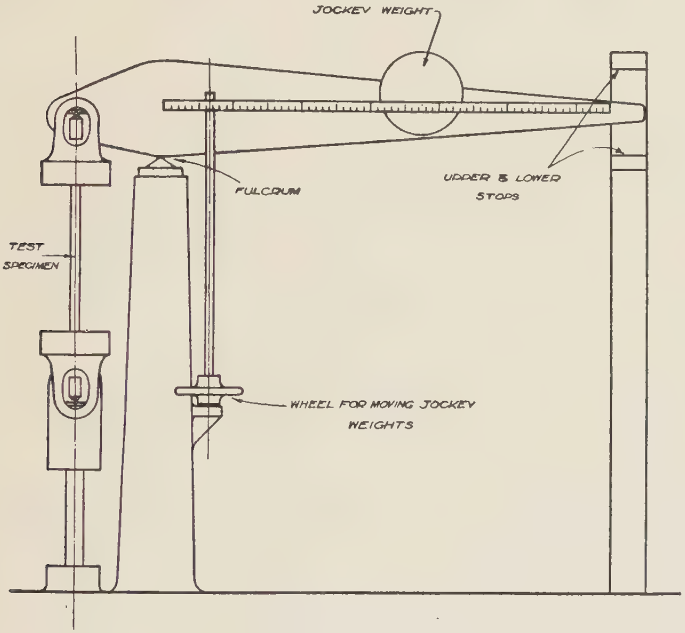


FIG. 2. Diagram of Testing Machine.

specimen by setting in motion the mechanism which pulls the lower grip downwards. This lifts the longer end of the beam, which probably touches the top stop.

The jockey weight is then run out (by turning the wheel which controls its motion) until the beam balances midway between the stops again, its position indicating on the scale the load on the specimen. More load can be applied and measured in a similar manner until eventually the specimen may be broken, the load required for this indicating, of course, the strength of the material of which it was made.

The student is strongly recommended to watch specimens of various kinds broken in a good testing machine as the nature

of materials and their method of failure is perhaps more clearly seen here than in any other circumstances. So accurate are modern testing machines that it is not unusual for a hundred ton machine to record accurately 5 lb.

Let us now proceed with our test. It is more instructive if, previous to fixing our specimen in the machine, we have made two indentations on it with a pointed centre punch about 8 in. apart near the middle. We adjust a pair of compasses so that its points will just go into both these punch-marks, and then clamp it. When we apply a pull on a specimen we can then put one end in one mark, and see, by comparing the position of the other with the other mark, how much the 8-in. length of specimen has elongated.

We first apply 5 tons in the manner already described. Our compass shows no stretch or elongation. We increase the load to 10 tons and 15 tons successively, and still our compass shows no stretch. There is, in fact, still no visible change on the specimen.

At about 17 tons (depending on the particular specimen) we notice a change. Up to this point only a few revolutions of the actuating mechanism (for applying load) were required to increase the load by 5 tons. Now we find that though we keep this mechanism running the jockey weight does not require running out to maintain balance. Our compass points indicate that the specimen is elongating, as they will not both enter the two punch-marks as before. If we continue running the mechanism which applies load to the lower end (by pulling it down), the stretch of the specimen increases, yet the jockey weight still remains almost stationary to keep the beam balancing. We may now notice the rod getting a little thinner at some parts.

After a time the continued stretch of the specimen requires the jockey weight to be run out to retain balance (in other words, the load on the specimen now increases), and the elongation and thinning of the rod increases. This thinning is generally local, one or more necks (or waists) appearing on the rod (Fig. 3 *b*).

Eventually the rod snaps at one of these necks with a considerable report, and the end of the beam drops on to its lower stop (which should have rubber or other soft packing to receive it). The load required to produce this fracture (called the *ultimate* or *breaking load*) can now be read off the scale. The specimen generally looks somewhat like Fig. 3 *c*, the fracture

generally presenting a grey and rough surface. Generally, one end is cupped, and the other has the complementary shape.

If we fit the two pieces together and use our compasses we can measure the amount the 8-in. length has stretched. This is called the *ultimate elongation.* It is generally expressed as a percentage of the original length measured.

British Standard Specification for mild steel requires this not to be less than 20 per cent. on a length of 8 in. ($\frac{20}{100} \times 8$ in. = 1·6 in.), and an elongation of 2 in. is not uncommon.

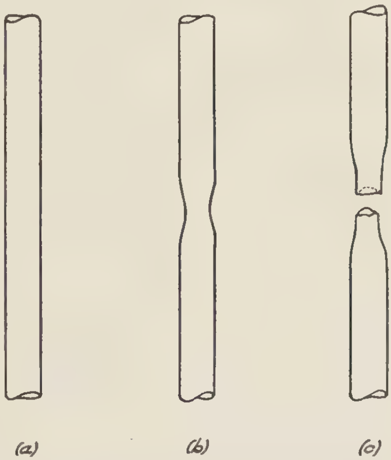


Fig. 3. Successive Stages of Failure in Tensile Test.

If we divide the ultimate load by the original cross-sectional area of the rod we obtain a figure which is called the *ultimate stress,* and may be expressed in tons per square inch. Other units of load and length may be used, such as lb. per square in., or tons per square foot, but a stress must always be expressed in units which state both the unit of weight and the unit of area. To talk of a stress of ten tons is as much nonsense as to talk of the distance from London to Brighton being so many square feet, and a student should be very careful to specify always the units he is using, especially when dealing with units (such as those of stress) with which he may be unfamiliar.

The British Standard Specification (B.S.S.) requires the ultimate stress for mild steel to be not less than 28 nor more than 33 tons per square inch.

If we measure the diameter of the rod at the fracture, and calculate the sectional area, we can compare this with the original area of section. The reduction of area at fracture is specified in the B.S.S. not to be less than 40 per cent.

The stress (generally about 17 tons per square inch) at which the specimen began to elongate visibly (with the help of a pair of compasses) is called the *yield point.*

The characteristics of a particular specimen of steel can be recorded if, during the experiment just described, we make

measurements of the elongation corresponding to various stresses. These measurements would be somewhat as follows :

Load.	*Visible Elongation.* in.
5 tons	0
10 „	0
15 „	0
17 „	0·75
20 „	1·05
24 „	2·1 [1]

Now, although those measurements are recorded as load in tons and elongation in inches it is convenient to calculate two cognate quantities which are closely related to them—namely, *stress* and *strain.*

¶ STRESS is the load per unit area. When (as in our case) the stress is uniform over the section it is calculated by dividing the load in tons by the original cross-sectional area in square inches, the result being the stress in tons per square inch.

¶ STRAIN is the elongation divided by the original length, in our case 8 in. So that, when the elongation is 0·75 in., the strain is

$$\frac{0{\cdot}75}{8} = 0{\cdot}094 \text{ approximately,}$$

and so on. Notice in passing that the unit of strain is one length divided by another length, which is a pure number or ratio, and quite independent of the units in which the lengths are measured.

Thus, though the elongation was 0·75 in. it would be nonsense to say that the strain was 0·094 *inches.* The strain, being the ratio of one length (0·75 in.) to another length (8 in.), is merely a fraction, and has no other unit.

The reason we prefer to think of stress rather than load is because it tells us the condition of the specimen without reference to its size or shape. If we are told the *pull* on a bar is 30 tons we cannot know anything about its safety unless we are also told whether it is 1 in. square or 2 in. diameter, or whatever it may be. In the first case the specimen would be near fracture ; in the second perfectly safe. On the other hand, if we know the *stress* is 30 tons per square inch then we know it is near the breaking point if the material is ordinary mild steel.

[1] Specimen broke. Elongation measured after fracture.

Similarly, the elongation gives us little indication of the condition of the material unless we are also told whether this elongation occurs in a length of 8 in. or in 8,000 in. (as may easily happen with a steel cable carrying the cage in a mine). In the one case a 2-in. elongation would indicate a condition near the breaking point; in the latter a perfectly safe condition.

If, however, we are told the *strain* is so much we know the whole story without bothering to inquire what the length is.

We can now convert our loads and elongations into stresses and strains respectively, in the first case by dividing by the area (area of 1 in. diam. rod is 0·78 sq. in.), and in the second by dividing by the original length (8 in.).

The results of our experiment may now be recorded as follows:

TABLE I

Load	*Stress: tons per sq. in.*	*Visible Elongation in.*	*Visible Strain in.*
5 tons	6·4	0	0
10 ,,	12·8	0	0
15 ,,	19·3	0	0
17 ,,	21·9	0·75	0·094
20 ,,	25·7	1·05	0·131
24 ,,	30·9	2·1	0·262

These results can be plotted on a piece of squared paper. We generally measure the stress vertically and the strain horizontally. For each value of the stress we measure along an horizontal length proportionate to the corresponding strain, and mark the point so arrived at. These are shown in Fig. 3 A as circles, corresponding to the figures in the preceding table, and a line drawn between these points gives us the characteristic stress curve for that particular steel.

These stress strain curves give quite a lot of information to an experienced person. In the first place they show clearly the yield point, the breaking point, and the ultimate strain, all important properties, which vary for different steels.

For example, the curve for another steel is also shown on Fig. 3 A by dotted lines. This would be a more brittle steel. Both its yield point and its breaking point are a little higher, but its strain is much lower. This steel has an ultimate strain hardly sufficient to pass B.S.S.

Generally speaking, with ordinary mild steels, increase of strength is accompanied by loss of strain or elongation. In-

crease of strength can be obtained by the addition of a greater percentage of carbon than is usual, but such increase results in loss of strain and increase of brittleness.

A material that breaks with a small strain is more brittle than one which can be extended a great deal before breaking. Extreme examples of brittle materials are cast iron and concrete. In both cases the ultimate strain would generally be less than 0·01 (1 in. in 100 in). Brittle materials have definite disadvantages as compared with yielding materials for con-

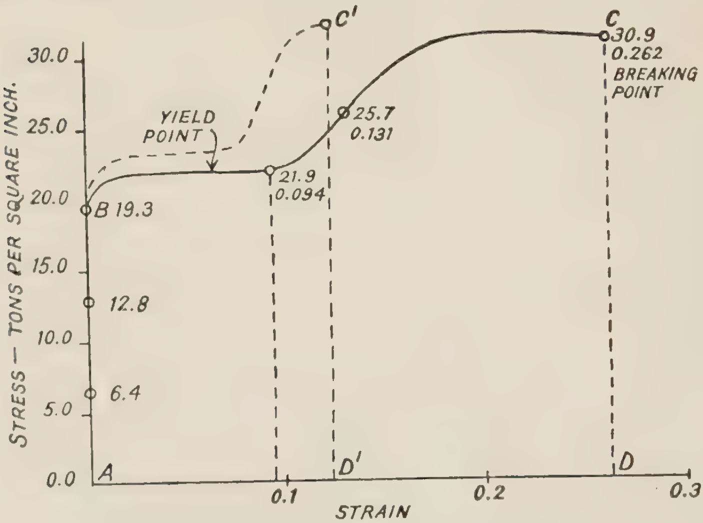


FIG. 3 A. Stress-Strain Curve for Mild Steel.

structional work. In the first place, when overstressed they fail suddenly without warning, whereas the yielding material gives warning during the period of yielding.

Secondly, in the process of fabricating girders and stanchions, and in the subsequent riveting on site, much hammering takes place which would be liable to break them were they made of brittle materials.

Thirdly, local overstressing, due to uneven contacts at joints, is largely relieved or mitigated by yield in a yielding material, but would cause fracture in a brittle material.

For these (and other) reasons engineers have found relatively soft or mild steel better and more reliable than harder or stronger steels, the property of yield being an important one.

It is possible in modern metallurgy to supply special steels

which combine a great strength without a corresponding loss of ductility, and for large bridges considerable use is made of such material. But for ordinary constructional work its increased cost has so far prevented its adoption, and it will not be referred to here.

The area under the stress strain curve (*ABCD* in Fig. 3 A) is a measure of the work done on the specimen before it breaks. Clearly the greater this is the less liable is a structural member to be broken in certain practical contingencies, such as by a falling weight, or by itself falling, where in both cases the energy available for producing fracture is limited, and depends on the magnitude of the falling weight and its speed.

It will be seen that with the more brittle (or less ductile) specimen whose curve was given by dotted lines in Fig. 3 A the area under the diagram (the area *ABC′D′*) is much less.

CHAPTER II

FACTOR OF SAFETY

FROM what has been said it will be clear that when steel is stressed to a definite point called the yield point, which varies somewhat with different specimens, but lies round about 17 tons per square inch for ordinary mild steel, it begins to flow or yield to a very appreciable extent, the load on the specimen remaining practically constant through this phenomenon. Consequently, if we reached this point in any practical structure, this structure would deform so much as to be quite useless. If a tie in a lattice girder were to stretch in this manner the girder would deflect so much that the riveted joints would be so twisted and distorted as to fail, and the loading on the stanchions or other supports would become so eccentric as to cause these to fail also, in all probability. In any case the deflection of the girder would become so great as to break every ceiling and floor dependent on it ; and, of course, to be quite useless for any architectural or structural purpose. If we are considering a specimen in compression which is liable to buckle we shall see presently that buckling will take place as soon as this yielding condition is reached.

It is, therefore, quite clear that we must so design our structures that the actual stress never approaches this yield point, and that for all practical purposes the yield point is really the absolute limit to which the stress can go in structural work before failing of the structure occurs. We must, in fact, differentiate between failure of the test specimen and failure of the structure.

It is true that our test specimen, after passing the yield point, had to be stressed to a much higher figure (somewhere about 30 tons per sq. inch) before it actually broke, but for the reasons already given a composite structure, such as a lattice girder, fails for all practical purposes when the yield point is reached, and this must therefore be regarded as the real danger point when we are dealing with structures.

It is convenient in practical design to agree on the number of tons per square inch which we can safely adopt in a practical design. This is called the *working stress* or the *safe stress*. In the case of mild steel this is generally taken at about $7\frac{1}{2}$ tons per square inch.

The *factor of safety* is generally defined to be the ratio between the breaking stress and the working stress. Where the breaking stress is 30 tons per square inch and the working stress is $7\frac{1}{2}$ tons per square inch the factor of safety would therefore be :

$$\text{Factor of safety} = \frac{30}{7\frac{1}{2}} = 4.$$

This nomenclature is most unfortunately chosen because it not unreasonably conveys the impression that if the factor of safety is 4 the structure could be loaded up to four times its weight before failure would occur. This is not by any means the case because, as has already been stated, failure would in all probability occur when the yield point was reached, and it would in many ways be much better if the factor of safety had been defined as the ratio between the yield point and the working stress ; but it has been defined so long in the other manner that it would, perhaps, at this stage only make confusion worse if the same words were now given a fresh meaning.

Let the student then be on his guard against this, and realize that a factor of safety of 4 merely denotes the ratio between the breaking stress of a test specimen and the working stress, and does not in any sense imply that the structure can be loaded to four times the load for which it was designed.

Assuming that the yield point of the particular steel used was 17 tons per square inch there would in fact be a margin of

$$\frac{17}{7\frac{1}{2}} = 2\frac{1}{4} \text{ approximately}$$

if a working stress of $7\frac{1}{2}$ tons per square inch had been adopted in the design. It is to be regretted that this figure instead of the previous one is not called the factor of safety, as it much more closely represents the actual margin of strength in the structure. Extensive tests on steel joists and girders of all kinds have shown that when these are designed with a factor of safety of 4—i.e. a working stress one-quarter of the breaking stress of the material—the structures themselves will only carry somewhere between $2\frac{1}{4}$ and $2\frac{3}{4}$ the load for which they were designed.

It may not unreasonably be asked why it should be necessary to have this margin between working stress of $7\frac{1}{2}$ tons per square inch and the yield point of somewhere about 17 tons per square inch. Obviously our structures would be more economical if we adopted a working stress approximating much more

closely to the yield point. The chief reason for this is that this margin is one which is based on experience as being generally desirable to produce safe structures. This margin has, in fact, to provide against a considerable number of contingencies, all of which are unfortunately more or less possible in any practical work.

These contingencies include the following :

(*a*) Repetition of stress.
(*b*) Faulty material.
(*c*) Faulty design.
(*d*) Overloading of structure.
(*e*) Bad workmanship.
(*f*) Bad erection.
(*g*) Corrosion.

Experience unfortunately indicates that however much care we adopt, while each of these can be minimized none of them can be entirely avoided under practical conditions, and if the margin provided to allow for them is reduced beyond a certain reasonable point, which can only be determined by practice, the inevitable result would be that the percentage of failure is far greater than is fortunately the case to-day.

Let us consider the various points in greater detail.

¶ Repetition of Stress. It has been found that a specimen which fails in the testing machine under a single application of 30 tons per square inch will fail, for example, at 25 tons per square inch if the load is applied a certain number of times. It will even fail at 20 tons per square inch, but the number of applications would be immensely greater. This fact is, of course, well known and taken advantage of by any one who bends a piece of iron backwards and forwards a great many times with the object of breaking it.

Clearly a structure like a warehouse is designed to be loaded and unloaded as frequently as one's clients' requirements may demand, and must therefore be designed at a stress so low that an infinite number of applications would not cause fracture.

A great deal of research work on this subject has been done, and testing machines are in use in many research laboratories, so arranged as to apply a load and remove it so rapidly that several million applications can be made in a reasonable time under conditions in which the number of times can be recorded by the machine. This has added a great deal to our knowledge of the subject.

It will be unnecessary to deal with the point in greater

detail in this elementary treatise except to draw attention to the point that it is one which the factor of safety is called upon to provide against.

¶ Faulty Material. Theoretically, of course, no faulty material should ever find its way into a job, and the vigilance of engineers undoubtedly prevents this to a very large extent. On all important work samples of the materials are tested from time to time, and any faulty material rejected. Nevertheless, it is impossible even for an engineer of great experience to say that no faulty material ever reaches the job, and if every time a piece of steel which had a yield point a few tons per square inch lower than that specified were incorporated in a large structure, and this were to produce a failure, then the number of failures would certainly be far greater than they are.

A considerable quantity of steel, for example, is imported from abroad. Even before the war foreign steel was often at a considerably lower price than British, and latterly the operation of the exchanges has, of course, accentuated this difference. It so happens also that French and Belgian specifications for steel are much lower than the British. The French specification, for example, only requires 24 tons per square inch ultimate, as against the British 28 to 33, and it requires great vigilance to ensure in the stress of practical working, when perhaps a client's revised requirements at the last moment have produced a demand for a section not originally contemplated, but which a contractor happens to have in stock in a foreign section and not in a British one, that in no single instance will any substitution ever be made.

Even in British material a slight excess of phosphorus or sulphur in the chemical constituents of the steel may easily produce a weakening from the standard specification which may not in every case be detectable.

The process of rolling whereby the ordinary commercial sections of angles, joists, &c., are produced from steel ingots also leaves these sections with residual stresses due to this deformation and to the unequal cooling of the different portions of the sections. The web of the joist being much thinner than the flanges naturally cools more rapidly and tends to shorten more than the flanges. This shortening obviously cannot take place, and a state of stress is, therefore, produced which does not entirely disappear when all the section has cooled down to normal temperatures. In some cases these stresses are so considerable that actual cracks or fractures are

produced in the sections. In this case that particular section would probably be thrown out, but there remain the other thousands of specimens where these initial stresses were not sufficient to cause the actual fracture, but nevertheless remain to a lesser extent in the section before it is incorporated in the structure.

Obviously the existence of these stresses weakens these members, and without going into the matter in greater detail here attention is merely drawn to the existence of these stresses as one of the matters which a factor of safety is required to cover.

¶ Faulty Design. There should never be faulty design, but in practice one is bound to admit that one frequently comes across it. How many stanchions, for example, which are actually eccentrically loaded, are designed as if they were concentrically loaded, the error in some cases easily amounting to 100 per cent. How frequently the stresses due to the structure wracking under the influence of wind or of subsidence of one or more of the points of support are unprovided for in the design, and the margin between the working stress and the yield point is the only margin provided to stand between the design and disaster.

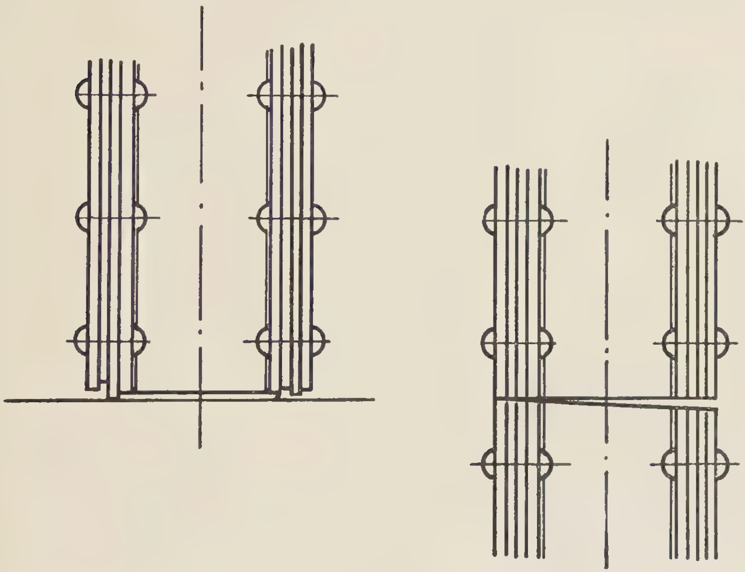
¶ Overloading of Structure. Clearly no structure should ever be overloaded, but how frequently have we not all seen a building, for example, designed to carry floors intended for a load of 100 lb. on a square foot, loaded up with a heap of bricks, sand, or other builder's material on the floors to a height of 3 ft., so producing an actual loading two or three times as great as that for which the structure is designed.

The reader will understand that we are not defending this practice. On the contrary, the vigilance of the engineer must be continually directed towards eliminating it as far as possible, which can be done in many ways, such as by a greater spreading of these necessary loads, by strutting such heavily loaded areas, and by many other methods. Nevertheless, one is bound to admit after a considerable experience that in spite of all such vigilance overloading to some extent, and at some periods, does take place in most of the buildings with which one is acquainted, not only during erection but also subsequently. How often has a comparatively light floor had a two-ton safe wheeled across it. These examples could be multiplied almost indefinitely.

No doubt one would be justified in adopting the attitude that theoretically none of these should ever be allowed to happen, but practically we know they do happen, and the factor of

safety does undoubtedly contain an element which provides for these cases.

¶ Bad Workmanship. When the writer first had to do with constructional steelwork in London it was the practice of more than half the steelwork firms to cut the steel joists and plates forming a length of stanchion, drill and rivet them together,



(a) (b)

Fig. 4. Examples of Bad Workmanship in Stanchion Joints.

and supply this to the job for erection without further effort. The consequence was that it was not an uncommon thing to find the end of a stanchion presenting the appearance somewhat as shown in Fig. 4 *a*, where the various plates and the joists would project beyond one another to varying extent. The upper end of the lower stanchion on which this would rest would present a somewhat similar appearance, and it would be entirely a matter of chance if there was any concordance between these irregular projections.

The net result would be that instead of the stanchion bearing uniformly over the whole cross-sectional area, as was assumed in the calculations, it would be much more likely that bearing would take place to the extent of only one plate on each flange, which might represent perhaps one-quarter of the

total cross-sectional area, or four times the stress allowed in the design.

It is true that the projecting plate, which really carries the load at the joint, would yield and shorten much more than the others, and might in certain cases do so sufficiently to bring some of the other plates into contact, but this relative movement could only take place if a partial shearing of rivets accompanied it, and it also involved a considerable yield or shortening of stanchions, which would only take place as the full load was approached. This would frequently mean that in a building of ten stories, for example, the joints near the bottom would begin to shorten up as the brick and stonework was approaching the roof, and the writer knows of many cases where fracture of stonework was directly due to this cause.

Fortunately there is now no constructional steelworks in London of any reputation which does not, as a matter of ordinary routine, machine the ends of the stanchions after the joists and plates have been riveted together, so as to produce a perfectly flat face. This is generally done either in a large lathe or in a special machine (called an ending machine), consisting essentially of a large revolving plate provided with cutting tools in the form of knives, against which the end of the stanchion is gradually brought up. Even with this precaution the possibility of bad workmanship is not eliminated, because the ends of the stanchions, though machine planed, may not be truly at right-angles to the stanchion, so that two lengths when erected may present the appearance shown in Fig. 4 *b*. The writer has had experience of stanchion joints of this kind, where the opening amounted to as much as $\frac{1}{4}$ in.

It is clear that in these cases instead of the whole stanchion sharing the load uniformly there will be a grossly excessive concentration of stress on the flange where contact is first made. In cases of this kind the expedient of wedge-shaped packing pieces has been used, and is better than nothing, but certainly will never reduce the stress down to the figure which would be obtained with perfect machining.

Fig. 5 shows a bearing of a beam on a stanchion flange bracket illustrating two faults. The first of these is the intensity of stress in the web of the beam and also in the web of the bracket stiffener owing to the short bearing. In calculating this stress the width of flange of the beam and of the angle bracket hardly affects the result, as it is the comparatively thin web through which the stress has to be transmitted, and here it

will be found that excessive stresses are produced unless a proper bearing is given. This may produce dangerous conditions, and in any case produces a yield which sometimes is responsible for cracking in the other materials of which a building is composed.

The second fault illustrated in this diagram is that the stiffener to the angle bracket does not touch the angle bracket

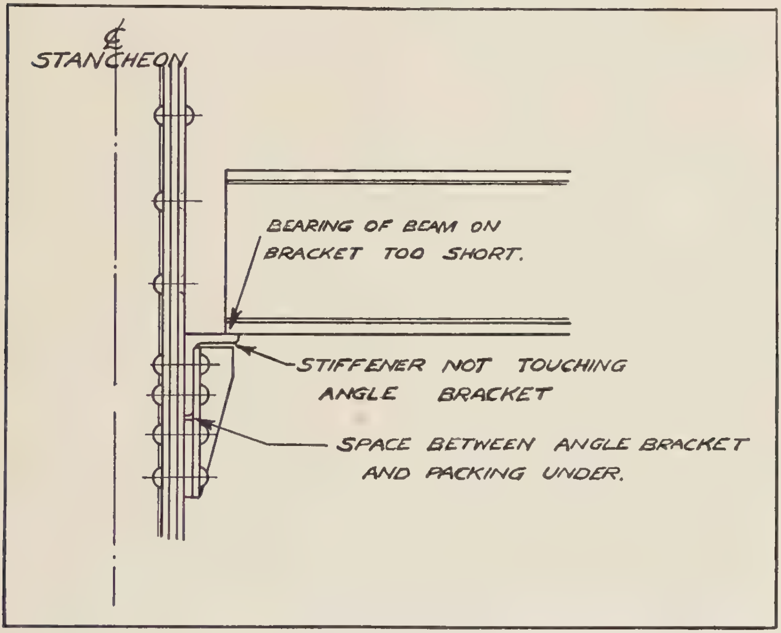


Fig. 5. Examples of Bad Workmanship in Beam Bearings.

at the top, with the result, again, that deformation and yield must take place when the load is applied.

A third fault is illustrated in the space between the angle bracket and the steel packing under it. The bracket is connected to the stanchion by four pairs of rivets, which are intended to share the load equally, but when executed as shown in the drawing the angle bracket would be forced down on to the packing plate under it so as to close up this space before the lower two pairs of rivets took their proper proportion of load. All this yielding and accommodation involves partial shearing of rivets and produces excessive local stresses. These examples could be multiplied almost indefinitely, but perhaps enough has been said to illustrate the point for the time being, and it will be referred to again later.

¶ Bad Erection. It sometimes happens that stanchions are put slightly out of position on their foundations, and are then pulled over by the floor-beams at the level of the next floor, these beams being to correct length and governing the distance between the stanchions at this level. The effect is almost the same as in Fig. 4 *b*, showing that bad erection can be just as dangerous as bad workmanship.

The base of a stanchion is intended to receive uniform bearing on the concrete under it. Where reliance is placed on grout the space under the base plate at the top of the foundation is filled with a mixture of cement and water, which is allowed to stand and ultimately sets. Experiments made by lifting stanchions off joints made in this manner have shown that though round the edges the cement fills the whole space (assuming a head of grout is provided by constructing a little dam round the joint and keeping the grout with a head above the joint), yet towards the middle of the joint the cement merely settles to the bottom and leaves water above it which cannot get away, and consequently is not replaced by cement, with the result that the stanchion base rests on a concave surface making contact round the edges and leaving a space which may amount to 1 in. or more near the centre.

Conditions of this kind throw upon the gusset plates more load than they were intended to carry, and also produce on the cement a much greater intensity of pressure than it was designed for. In both cases the result is a considerable overstressing of materials and a yield and dropping of the stanchion when the load is applied, which may produce cracks in a building not unlike those due to unequal settlement of foundations. This example was specially given because it is one for which the steel contractor may not be responsible.

It may be mentioned in passing that in the writer's practice grouting has for many years been given up entirely, and the space under stanchion bases is filled by ramming a stiff mortar mixed up with the minimum of water so that the space is entirely filled with a material which is not subject to appreciable contraction subsequently, and which also has much greater load-carrying capacity. Examples of bad erection could, of course, be multiplied almost indefinitely.

¶ Corrosion. The degree to which steelwork is subject to corrosion varies, of course, tremendously. In the case of roof trusses over railway stations, steelwork exposed in industrial areas containing sulphuric acid, nitric acid, and other corrosive

elements in the atmosphere, steel exposed in hot, damp climates such as are met in many tropical countries, corrosion may be so rapid that a definite thickness of material is allowed all round as a specific provision for corrosion, and the structure need then not be considered as less strong than as designed until this allowance for corrosion has been exceeded.

Where, however, no such allowance is made (and this is not ordinarily the custom), then any corrosion has got to be provided for out of the margin allowed for the factor of safety. It is true that on exposed structures much can be done by thorough and frequent painting, but there are some atmospheric conditions in industrial areas where paint is destroyed very rapidly. The writer knows of one case where within the six months' period of maintenance the whole of the exposed steelwork had thrown off every vestige of paint, though this had received four coats of first-class material put on in the best manner.

In steel frame buildings the steel is practically always covered, partly as a precaution against fire and, in many cases, so as not to shock architectural susceptibilities. In such cases painting is, of course, impossible, at any rate after it has once been covered up. If the steel is thoroughly protected with two inches of really watertight and airtight cement mortar, or concrete, the precaution against corrosion will generally be entirely adequate for as many years as are generally contemplated in modern building work. But the writer knows of many cases where a steel frame structure has had brickwork built hard up against the steel. Remembering that ordinary stock brickwork contains about 30 per cent. of voids allowing passage of water and air to the steel, it will be realized that under these conditions something serious is going to happen in a small number of years. In the writer's view it is no good under these circumstances relying on paint on the steelwork, because paint is rapidly destroyed by cement mortar, and lime, the linseed oil being saponified by the alkalies.

Much more serious attention to the question of corrosion of steelwork in steel frame buildings should be given than is generally done, and the future will present interesting problems in the case of those buildings where these questions have been allowed to look after themselves. When this time comes it will, at any rate, be some small satisfaction to know that the margin provided by the factor of safety will allow of a certain degree of corrosion before the structure is definitely rendered unsafe.

CHAPTER III

PROBLEMS OF ELASTICITY

IN the experiment on the testing of a mild steel rod in tension which we have described it was stated that there was no *visible elongation* up to the *yield point.* From what has been said it is clear that this covers the whole of the useful range of stress in structural work, and therefore we should be well advised to study our material through this range a little more carefully.

For this purpose we make use of an *extensometer*, an instrument for measuring very minute extensions which are too small to be visible to the naked eye. Extensometers are of various forms. All contain means whereby the specimen is clamped at one section, and then by a second clamp at a second section 8 in. away, movement between the two being measured in the case of *Unwin's extensometer* by rotation of a fine screw controlled by a delicate spirit level, and in the case of *Ewing's extensometer* by an optical device. We need not here be concerned with the details of construction, but it will suffice to know that a good extensometer will read to one ten-thousandth of an inch.

If we now repeat our test on a second specimen of mild steel rod through the lower range of stresses with our extensometer fixed in position, we should obtain results somewhat as follows :

TABLE II

Extensometer Tests on Steel Rod in Tension.

Load in tons.	*Stress : tons per sq. in.*	*Extension : inches on 8 in. length.*	*Strain.*
0	0	0	0
5	6·4	0·0038	0·00048
10	12·8	0·0076	0·00096
5	6·4	0·0038	0·00048
0	0	0	0

The actual readings are, of course, those recorded in columns 1 and 3, the first being given on the beam of the testing

machine, and the second on the extensometer. The figure in the second column is obtained by dividing the load by the cross-sectional area of the rod (0·78 sq. in.), while the strain (in column 4) is obtained by dividing the elongation by the original length (8 in.).

The table shows that we have loaded our specimen up to 10 tons (12·8 tons per sq. in.) by stages of 5 tons, and then unloaded it by the same stages, taking readings at each loading both ascending and descending.

The results of the experiment (which are very important) may be summarized as follows :

(*a*) Although up to the yield point the elongation is not visible without delicate apparatus, yet there is in fact a definite elongation corresponding to the tensile stress.

(*b*) This elongation increases proportionately with the stress, in other words, *within the elastic limit the strain is proportional to the stress.*

(*c*) When the load is removed the specimen returns to its original length. In other words, *when the stress is kept below the yield point there is no permanent yield or permanent set,* and the material is as if it had not been loaded.

The foregoing results of our experiments can be expressed in simple mathematical form, as follows :

$$\frac{\text{Stress}}{\text{Strain}} = E \quad \text{.........................} \quad (1)$$

where E is a constant. This constant is called *Young's Modulus of Elasticity,* and is a constant for any material over the whole range up to the yield point.

For steel this constant has the value of 30,000,000 lb. per sq. in. approximately, and for concrete it varies between 2,000,000 and 4,000,000 lb. per sq. in., being more for the stronger concretes than for the weaker ones.

This relationship is a very important one and is known as *Hooke's Law.* It obviously enables us to calculate what the elongation will be on a material of known section under the action of a known tension. Conversely it obviously enables us, by an observation of the elongation in a material within the elastic limit, to state what was the tension which produced this elongation.

Example I. A tension member of steel 30 ft. long, 3 in. by 1 in. in section is subject to a tension of 20 tons. What will be the elongation ?

Answer. Equation I may be rewritten as follows :

$$\text{Strain} = \frac{\text{stress}}{E}$$

$$= \frac{\dfrac{20 \times 2{,}240 \text{ lb.}}{3 \text{ sq. in.}}}{\dfrac{30{,}000{,}000 \text{ lb.}}{1 \text{ sq. in.}}}$$

$$= 0{\cdot}0005.$$

$$\begin{aligned} \text{Therefore elongation} &= \text{strain} \times \text{original length} \\ &= 0{\cdot}0005 \times 360 \text{ in.} \\ &= 0{\cdot}18 \text{ in.} \end{aligned}$$

Example II. A bridge member has the elongation on an 8-in. length measured by means of an extensometer, and is found to give an elongation in 8 in. of 0·004 in. If the section of the bridge member is 12 in. by 1 in., calculate the tension which produces this elongation.

Answer.

$$\text{Strain} = \frac{\text{elongation}}{\text{original length}}$$

$$= \frac{0{\cdot}004 \text{ in.}}{8 \text{ in.}}$$

$$= 0{\cdot}0005.$$

Therefore the stress

$$\begin{aligned} &= \text{strain} \times E \\ &= 0{\cdot}0005 \times 30{,}000{,}000 \text{ lb. per sq. in.} \\ &= 15{,}000 \text{ lb. per sq. in.} \end{aligned}$$

Therefore tension

$$\begin{aligned} &= \text{stress} \times \text{sectional area} \\ &= 15{,}000 \text{ lb. per sq. in.} \times 12 \text{ sq. in.} \\ &= 180{,}000 \text{ lb.} \end{aligned}$$

One of the important properties of structures which are not stressed beyond the yield point is that when they are unloaded they revert to their original condition before loading. A girder, for example, which is deflected under load goes back to its original position when the load is removed, but if it is loaded beyond the yield point it will only partially return on the load being removed. Part of the deflection in that case is a permanent one and part an elastic one, and when the load is removed the elastic deformation disappears, but the permanent deformation remains. This is one of the tests which are applied

to structures in practice, to load them up and to note the deflection.

If when the load is removed the deflection does not disappear this may be an indication that the structure has been loaded beyond the yield point, in which case it will be clear that it is probably in a dangerous condition, or at any rate that so great a load cannot be put upon it and removed again an indefinite number of times without producing failure.

CHAPTER IV

BENDING MOMENTS

IN studying the problems connected with bending it is desirable to make an early acquaintance with two quantities called *bending moments* and *moments of resistance* respectively, the use of which facilitates the work very much, as we shall see later. For the sake of any readers who are not familiar with these conceptions (or may desire to review their knowledge from perhaps a slightly different angle) they will be explained in the simplest possible terms.

The idea underlying the bending moment is that of leverage, of which every one has practical knowledge, even though this knowledge is frequently not one which could be put into words, and consists essentially of the conception that for many purposes it is not so much the magnitude of the force employed which produces results as the magnitude of a force multiplied by a distance or by a leverage.

Take, for example, the simple lever shown in Fig. 6, which is balanced at the fulcrum F and supports at its two ends weights W_1 and W_2. Assuming that the weight of the lever is negligible we find that equilibrium is only obtained when $W_1 \times l_1$ is equal to $W_2 \times l_2$.

By adjusting the distances we can make a 1-lb. weight balance a 10-lb. weight, and we see that the ability to depress one side of the lever depends not on the weight alone and not on the distance alone, but on a quantity which is the product of the two. This quantity is called a *moment* or a bending moment and may be defined as the product of a force into its distance. Thus, a moment of W_1 about F is $W_1 \times l_1$. The moment of W_2 about F is $W_2 \times l_2$, and so on.

We need not consider moments about the point F, we can consider moments about any point whatsoever. If we choose, for example, the right-hand side of the lever, then the moment of W_1 would be $W_1 \times (l_1 + l_2)$, and the moment of F would be $F \times l_2$ if F is the magnitude of the reaction at the fulcrum.

With moments it is important to consider the sign. Thus, the moment of W_1 about F is one which is tending to produce rotation in a direction indicated by the arrow, while the

moment of W_2 about F is obviously tending to produce rotation in the opposite direction.

It is therefore convenient to designate one of these as positive and the other as negative. Which is chosen as positive is purely a matter of convention, and in practice the counter-clockwise moments are generally considered to be positive and the clockwise moments negative.

We see that at the point F we have a clockwise moment of $W_2 l_2$ and an anti-clockwise moment of $W_1 \times l_1$ which is equal to $W_2 l_2$ but positive in sign.

This is an illustration of the general law that *in a stable structure the sum of all the moments about any point is zero*, or, put otherwise, *the sum of the clockwise moments about any point shall balance the sum of the anti-clockwise moments.* It is perhaps instructive to notice that this does not apply only to the point F.

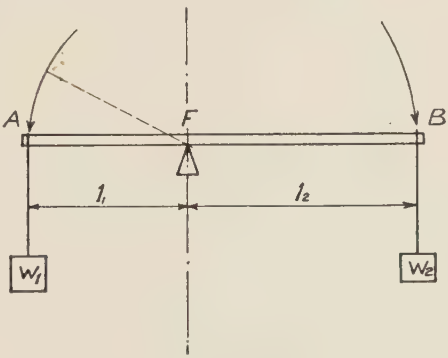


FIG. 6. Simple Lever to illustrate Bending Moments.

Suppose, for example, we consider moments about the point A. The force W_1 obviously has no moment about this point, because as it comes through the point its distance from it is zero. There are therefore only two forces producing moments, viz. the force F and the force W_2. The magnitude of the force F is clearly the sum of the weights, viz. $W_1 + W_2$, since the whole weight of the beam and the weights hanging from it are carried on the fulcrum at this point. The moments about A may therefore be written as follows :

Anti-clockwise, $F \times l_1$.

Or substituting $F = W_1 + W_2$,

this moment may be written

$$(W_1 + W_2) \times l_1 = W_1 l_1 + W_2 l_1.$$

Instead of $W_1 l_1$ we can substitute $W_2 l_2$ so that our moment may be written

$$W_2 l_2 + W_2 l_1 = W_2(l_1 + l_2).$$

Now the moment of W_2 about A is clearly W_2 multiplied by its distance about A, which is (l_1+l_2), so that this moment may be written

$$W_2(l_1+l_2),$$

and being in the opposite direction to the moment exerted by the force F would be written with an opposite sign. Hence the sum of the moments about A is

$$W_2(l_1+l_2)-W_2(l_1\times l_2)=O,$$

illustrating the fact that in a stable structure the sum of the moments *about any point* is equal to zero.

If about any point it can be shown that there is a moment which is not balanced then this moment will produce rotation of the structure about this point. It is highly desirable that in buildings and similar structures there shall not be rotation about any point.

Similarly in regard to the forces acting. In the example given the upward force F is exactly equal in magnitude to the sum of the downward forces W_1+W_2, and is an illustration of the general law that *in a stable structure the sum of all the forces in one direction is equal to the sum of all the forces in the opposite direction.* If the forces in one direction are called positive and those in the opposite direction negative then this can be expressed by stating that *the algebraic sum of all the forces in any direction in a stable structure must equal zero.* If in any structure it can be shown that the forces in any direction do not balance with those in the opposite direction, then the unbalanced force produces motion or translation in the direction of that force.

In our example if F were greater than the sum of the loads on the lever the whole system would be moved upwards, and conversely if the weights on the lever acting down were greater than the upward reaction at F then the whole structure would move in a downward direction.

It is highly desirable in buildings and other stable structures that there should not be any unbalanced forces producing motion.

These two laws, though so exceedingly simple and obvious once they are stated, often enable us to get a much clearer insight into the forces acting on a structure and may be known as the primary laws of stable structures. They may perhaps best be written as follows :

Law 1.—The sum of all the forces in a stable structure in any

direction must balance the sum of the forces in the opposite direction.

Law 2.—The sum of all the clockwise moments in a stable structure about any point must balance the sum of all the anti-clockwise moments about that point.

As an example of the use of these two simple laws in helping us to calculate the forces acting in a structure let us take the case illustrated in Fig. 7, where a rigid frame represented by two vertical members and one horizontal one is subject to the reaction of wind (represented by the force W acting at a distance l above the ground) and of a downward weight represented by P acting in the centre.

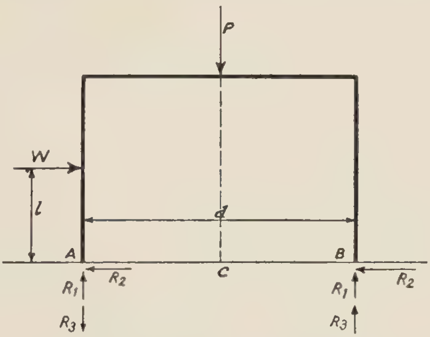


FIG. 7. Reactions of a Rigid Frame Subject to Wind and Weight.

Considering first the force P, it is clear that owing to symmetry it produces an equal upward reaction at each of the points A and B, and since the sum of the vertical forces in the upward direction must equal the sum of the forces in the downward direction (from Law 1), it is clear that each of these reactions, which we will enumerate R_1, must equal one-half of P, since

$$R_1 + R_1 = P.$$

Consider now the action of the wind force W acting horizontally.

If there were no other horizontal forces acting on the frame this wind pressure would set the frame in motion in a lateral direction, and as in a stable structure we know that this does not happen we have to find such horizontal forces. The only places where they can act are where the frame rests upon the ground, since in no other position are there solid materials in contact with the frame capable of exerting any horizontal forces upon it. If, as is occasionally the case, one of the supports is fixed and the other provided with a roller bearing then the whole of the horizontal reaction will be resisted at the fixed bearing and practically none of it at the roller bearing (assuming it to be practically free from friction). More frequently,

however, both bearings are equally fixed and share the hori zontal reaction equally. This horizontal reaction is represented on the diagram by the forces R_2, and applying Law 1 to the horizontal forces it is clear that

$$W = R_2 + R_2. \quad \text{Hence } R_2 = \frac{W}{2}.$$

The reactions R_1 and R_2 which we have calculated so far are, however, not sufficient to satisfy the requirements of the case, as will be clear at a glance if we consider the moments about the point C (the intersection of the vertical centre line with the ground). About this point the moment of the external forces is Wl, since P passing through this point exerts no moment about it. This moment is at present unbalanced, since neither of the reactions R_2 exerts any moment about C as they pass through it, while the reactions R_1 about C are obviously balanced in themselves.

It is in fact obvious that the force W exerts a lifting tendency at the point A and an opposite tendency at the point B, and to resist these tendencies a downward pull must be provided at the point A and a corresponding upward pull at the point B, these being denoted by the reactions R_3 on the diagram. These forces must be equal though opposite, since if they were not equal they would provide an unbalanced vertical force which would produce motion in an upward or downward direction, since the other vertical forces P, R_1 and R_1 are already balanced among themselves. For the purpose of valuating the magnitude of the force R_3 we may take moments about the point A. The moment of the wind force is Wl in a clockwise direction, and this is resisted by the reaction R_3 acting at a distance of d from the point A, producing a counter-clockwise moment of $R_3 \times d$, hence

$$Wl = R_3 d \text{ or } R_3 = \frac{Wl}{d}.$$

It is a simple matter to compound the reactions R_1, R_2 and R_3 into a single force at each of the points A and B, and it will be seen that these forces vary in direction and in magnitude.

The example illustrates the use of these simple laws in determining the forces acting on a structure.

Having been introduced to the conception of *bending moments*, based on the idea of *leverage*, and to the idea that the bending effect is due to the magnitude of the force multiplied by its leverage, we may now profitably consider the bending moments in a few commonly met examples.

Case 1. A CANTILEVER (*a beam built into a wall one end and unsupported the other*) *with a concentrated load at the free end.*

From the definition of a moment (the force multiplied by its distance from a given point) it is clear that the moment of W about A is Wl where l is the distance of W from A. This may be written

$$M_A = Wl,$$

M_A being the moment at A.

Similarly the moment at the point B (at a distance of $\frac{l}{2}$ from W) is clearly

$$M_B = W \times \frac{l}{2} = \frac{Wl}{2}.$$

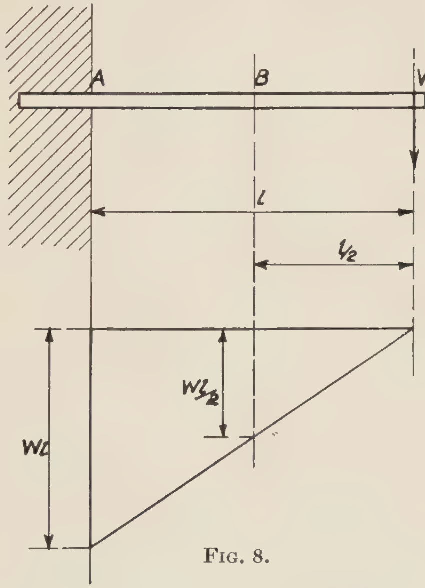


FIG. 8.

The moment about the end where the load W is suspended is clearly zero, because the moment is the product of the load into the distance from the load to the point considered, and where the point lies in the line of action of the load its distance from it is clearly zero.

The diagram in Fig. 8 shows how the moment varies along the beam, and is drawn by setting down a distance proportional (to some definite scale) to Wl under the point A, $\frac{Wl}{2}$ under the point B, and zero under the point W. The line joining the three points so determined is clearly straight, showing that the *distribution of moment of a concentrated load along a cantilever is a linear one having its maximum* (*equal to* Wl) *at the support.*

Case 2. A CANTILEVER *with a uniformly distributed load* W (Fig. 9).

This is the same case as the last except that the load is uniformly distributed, as it might be in the case of a floor loaded with a layer of sacks uniformly distributed.

It is clear that the load W does not act at a distance of l about the point A. Part of it (at the unsupported end) does, but the part near A acts at almost no distance. Clearly the average distance of the load from A is $\frac{l}{2}$ the distance of the centre of W from A.

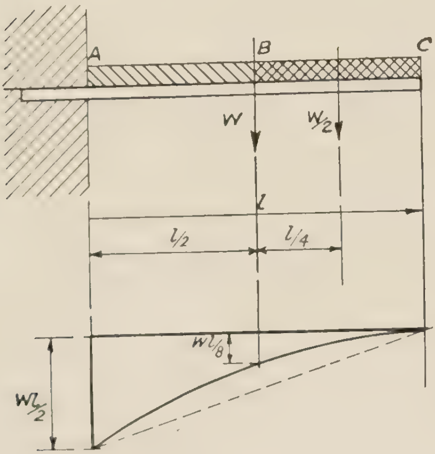


Fig. 9.

Hence the moment about A is

$$M_A = W \times \frac{l}{2}$$
$$= \frac{Wl}{2}.$$

If we consider the moment at the point B it is clear that only the portion of the load between B and C tends to produce rotation about B. Clearly the magnitude of this load is half the total load (i. e. $\frac{W}{2}$) and its centre acts at a distance of $\frac{l}{4}$ from B (see Fig. 9).

Hence the moment at the point B is

$$M_B = \frac{W}{2} \times \frac{l}{4} = \frac{Wl}{8}.$$

The moment at the point C is zero because there is no load to the right of C to produce any moment. Loads to the left of C cannot do so, because (from Law 2, see p. 35) in a stable structure a moment cannot exist unless it is balanced, and it would require loads to the right of C to balance loads to the left of C.

If we set down distances proportioned to $\frac{Wl}{2}$, $\frac{Wl}{8}$, and zero respectively, under points A, B, and C, and join them by an even curve we get the *bending-moment diagram*, showing how the moment varies along the beam with this particular loading. It will be seen that the diagram is not a straight line this time.

Clearly the moment at B would need to be half the moment at A for it to be so—actually it is a quarter. The curve, in fact, is a *parabola*.

Case 3. A BEAM *of span l simply supported at its ends, and loaded with a single concentrated load at midspan.*

If we examine half this beam, the part between *B* and *C* (Fig. 10) for example, we see it is exactly the same as the cantilever in Fig. 8, except that the load at the right-hand end now acts up instead of down, the beam at *B* being fixed, not in this case by being built in, but by having the upward load at *C* balanced by the upward load at *A*.

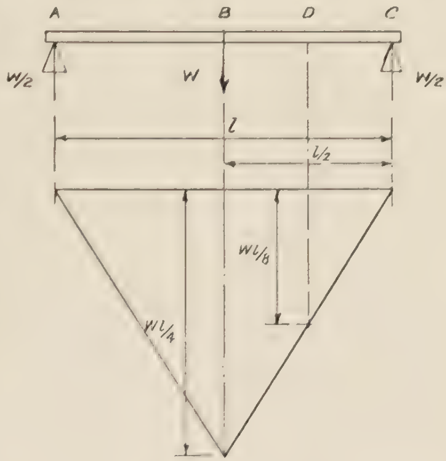


FIG. 10.

We first have to determine the forces at *A* and *C*. From Law 1 (p. 34) it follows that they must balance the only downward load W, and as from symmetry they are clearly equal, they must each be an upward force of magnitude $\frac{W}{2}\cdot$

Clearly the moment of *C* about *B* is

$$M_B = \frac{W}{2} \times \frac{l}{2} \text{ (see Fig. 10)}$$

$$= \frac{Wl}{4}.$$

We could have, had we preferred, obtained the same result by considering the moment of *A* about *B*, as the moments about *B* must balance or rotation takes place (see Law 2).

The moment at the quarter point *D* is obtained by multiplying the force at *C* by its distance from *D*:

$$M_D = \frac{W}{2} \times \frac{l}{4}$$

$$= \frac{Wl}{8}.$$

This being half the moment at *B* shows that the moment diagram is a straight line from *B* to *C*. It is given in full in

Fig. 10. It has its maximum value of $\frac{Wl}{4}$ at midspan, zero at the two supports, and varies uniformly between.

Case 4. A Beam *simply supported at its ends with a uniformly distributed load W on a span l.*

For the same reasons as in Case 3 the upward reactions at the supports A and C are $\frac{W}{2}$.

To obtain the moment at any point such as B (midspan) we have to consider the moments of all the forces on one side. We need not consider those on the other because they merely serve to balance them.

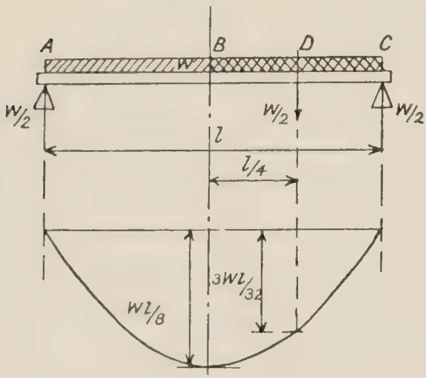


Fig. 11.

The forces on the right of B are:

(*a*) The upward reaction $\frac{W}{2}$ of the support C;

(*b*) The downward weight of the double-hatched part of load (see Fig. 11) which clearly is equal to half the total load (i.e. $= \frac{W}{2}$).

These tend to produce rotation in opposite directions about B, the moments being

(*a*) $\frac{W}{2} \times \frac{l}{2}$ anti-clockwise;

(*b*) $\frac{W}{2} \times \frac{l}{4}$ clockwise.

The moment at B is therefore the difference, so that

$$M_B = \frac{W}{2} \times \frac{l}{2} - \frac{W}{2} \times \frac{l}{4}$$
$$= \frac{Wl}{8}.$$

In exactly the same way, the moment at the quarter point D can be calculated, and will be found to equal $\frac{3}{32} Wl$, showing

that the moment diagram between B and C is not a straight line, but a *parabola.*

This moment diagram is given in Fig. 11, and is seen to have a maximum value of $\frac{Wl}{8}$ at midspan, and to die off parabolically to zero at the two supports.

These moment diagrams are of great importance in structural work, because they enable us to read off the moment at *any* section along a beam without fresh calculation. They also show us at a glance where the beam needs to be strongest and how the strength may be reduced at other sections. The simple moment diagrams and the values of the moments should be made so familiar that they can be written down or drawn at once without reference to any book.

Case 5. A BEAM *simply supported at its ends with various concentrated loads at various distances along it, not necessarily symmetrical.*

This case is perhaps most easily explained by taking at once a definite numerical example which will make the principle clearer than if it were explained in general algebraic terms, which frequently make a thing look more complicated than it is to a person not accustomed to them. Accordingly, in Fig. 12 an example is given where on a span of 20 ft. are loads of 4 tons, 5 tons, 6 tons, and 8 tons, the distances from the left-hand support being 4 ft., 10 ft., 15 ft., and 18 ft. respectively.

This case differs from those that we have previously considered in that the arrangement of loading not being symmetrical it does not follow that the upward reactions will each be equal to one-half the downward load, and before we can proceed we must calculate what these upward reactions are. Let us letter points A to F as in the diagram for convenience. If we call the reactions R_1 and R_2 for convenience, their values can be most easily calculated by applying Law 2 (p. 35), which states that the moments in a clockwise direction about any point must equal the moments in an anti-clockwise direction about the same point for the structure to be stable, and we select for our point most conveniently the point A, because the one reaction R_1 which passes through A clearly exerts no moment about it, and we are therefore left with a single unknown quantity, viz. R_2, in our equation, as will be seen in what follows.

Taking moments about A the clockwise moments are clearly as follows :

$$
\begin{aligned}
4 \text{ tons} \times 4 \text{ ft.} &= 16 \text{ ft. tons.} \\
5 \text{ tons} \times 10 \text{ ft.} &= 50 \text{ ft. tons.} \\
6 \text{ tons} \times 15 \text{ ft.} &= 90 \text{ ft. tons.} \\
8 \text{ tons} \times 18 \text{ ft.} &= 144 \text{ ft. tons.} \\
\textit{Total clockwise moments} &= 300 \textit{ ft. tons.}
\end{aligned}
$$

The only anti-clockwise moment about the point A is produced by the reaction R_2 so that :
total anti-clockwise moments $= R_2 \times 20$ ft. Since the clockwise and the anti-clockwise moments balance, we have

$$R_2 \times 20 \text{ ft.} = 300 \text{ ft. tons.}$$

$$\text{Whence } R_2 = \tfrac{300}{20} = 15 \text{ ft. tons.}$$

We can calculate R_1 by exactly the same procedure by taking moments about the point F, but it is simpler to make use of Law 1, which states that the sum of the upward forces must equal the sum of the downward forces. The downward forces are equal to 23 tons, and hence the upward forces ($R_1 + R_2$) are equal also to 23 tons, and since one of them, R_2, is equal to 15 tons it is clear that the remaining one, R_1, must equal 8 tons.

We can now calculate moments at various distances along the beam quite simply.

The moment at A is clearly zero, since there are no forces to the left of A.

The moment at B is clearly given by the reaction R_1 multiplied by its distance from B :

$$
\begin{aligned}
M_B &= R_1 \times 4 \text{ ft.} \\
&= 8 \text{ tons} \times 4 \text{ ft.} \\
&= 32 \text{ ft. tons.}
\end{aligned}
$$

The moment at C may be obtained by calculating the moments on the left-hand side of C. We have R_1 at a distance of 10 ft. producing a clockwise moment, and 4 tons at a distance of 6 ft. producing an anti-clockwise moment. The moment at C is clearly the difference between the two, so that

$$
\begin{aligned}
M_C &= 8 \text{ tons} \times 10 \text{ ft.} - 4 \text{ tons} \times 6 \text{ ft.} \\
&= 80 \text{ ft. tons} - 24 \text{ ft. tons.} \\
&= 56 \text{ ft. tons.}
\end{aligned}
$$

It is clear that to calculate the moment at a point such as C it is only necessary to consider the forces on one side, because the forces on the other side merely go to produce the balancing

and equal moment, and if we took the forces on both sides they would, of course, give us a moment equal to zero, as we know that the algebraic sum of the moment of all the forces about any point is zero. It therefore becomes simply a matter of convenience whether we use the forces on the right or the left, and in this case we use those on the left in preference to those on the right simply because they are less numerous.

We could, however, have used those on the right, and we should have obtained the same result. Perhaps it would be worth while to demonstrate that this is so. Taking, then, the moments of the forces on the right of C about C, we have in an anti-clockwise direction

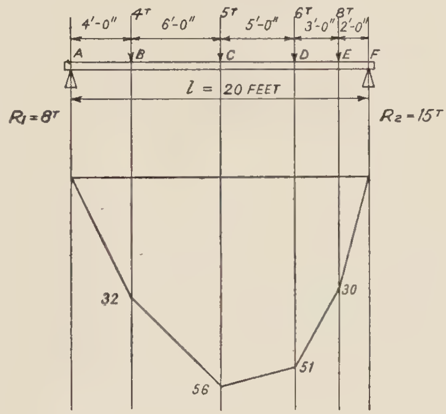


FIG. 12.

$$15 \text{ tons} \times 10 \text{ ft.} = 150 \text{ ft. tons},$$

and in a clockwise direction

$$6 \text{ tons} \times 5 \text{ ft.} = 30 \text{ ft. tons},$$
$$8 \text{ tons} \times 8 \text{ ft.} = 64 \text{ ft. tons},$$

so that

$$M_C = 150 - 30 - 64$$
$$= 56 \text{ ft. tons},$$

giving, of course, the same result as before.

The moment about D (taking the forces on the right of D) is given by
15 tons $\times$ 5 ft. anti-clockwise $-$ 8 tons $\times$ 3 ft. clockwise, so that

$$M_D = 75 - 24$$
$$= 51 \text{ ft. tons.}$$

The moment about E is clearly 15 tons $\times$ 2 ft., so that

$$M_E = 15 \text{ tons} \times 2 \text{ ft.}$$
$$= 30 \text{ ft. tons.}$$

The moment about F is, of course, zero, for the reason already given. By setting down these moments under the corresponding points to some definite scale we produce the bending

moment diagram given in Fig. 12. If these points are joined up by straight lines they give the moment diagram for this case, which may be used by scaling to give the moment at any intermediate point, though any such moment could easily be calculated by taking moments of the forces about it, exactly as we have already done about the points A, B, C, D, &c.

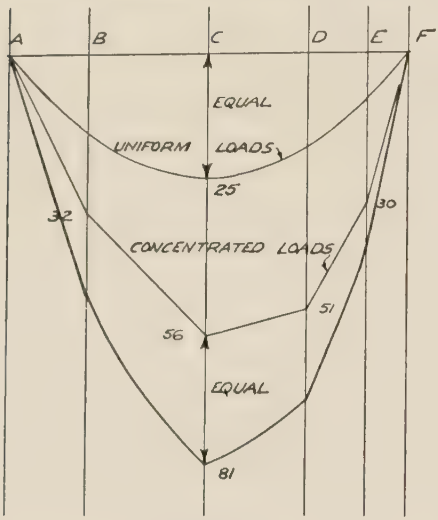


FIG. 13.

Case 6. *Combination of loadings on the same beam.*

It may easily happen that we get combinations of loadings on the same beam. For example, the beam which we worked out under Case 5 might, in addition to these concentrated loads, have been subject to its own weight of 10 tons uniformly distributed. In such a case it does not matter whether we calculate the moments separately and add them together or whether we write down all the forces on the same diagram and calculate the moments in a single operation, but generally the former procedure is easier and more convenient.

It will be clear that the moment at midspan of 10 tons uniformly distributed on a span of 20 ft. is

$$M = \frac{Wl}{8} \quad \text{(see Case 4)}$$

$$= \frac{10 \text{ tons} \times 20 \text{ ft.}}{8}$$

$$= 25 \text{ ft. tons.}$$

This produces a parabolic bending-moment diagram similar to that in Fig. 11, which can be drawn on the bending-moment diagram due to the concentrated loads as shown in Fig. 13.

The two diagrams, that due to the concentrated loads and that due to the uniform loads, can be left as separate diagrams, in which case if it is required to ascertain the moment at any particular point along the beam, the moment due to each can

be separately scaled and added together. It is generally more convenient to construct a diagram to which this addition has already been done, so that only a single scaling is necessary. This is done in the lower diagram in Fig. 13, which is prepared by setting down below the diagram for concentrated loads a length equal to the ordinate obtained from the diagram of uniform loads. The resultant diagram will be a figure with kinks at the points *B*, *C*, *D*, *E*, where the concentrated loads occur, and will not be quite straight between these points, as clearly a slight curvature is induced by the curved diagram for uniform loads which has been superimposed on the straight diagram.

There is an interesting simile between bending-moment diagrams and the shape taken up by a piece of string when its ends are secured and loads are suspended from it in places corresponding to those on a beam whose bending-moment diagram is to be studied, which when once grasped makes it much easier to realize what the form of a bending-moment diagram should be in any given case, and which should therefore commend itself to students. For example, it was already shown in Fig. 10 that the moment diagram of a beam carrying a single concentrated load at midspan was a triangle, and it is clear that a load suspended at midspan from a string tied at the two ends would take up exactly that shape (see Fig. 14 *a*).

Similarly a beam subject to uniform loading was shown in Fig. 11 to produce a parabolic bending-moment diagram, and it is, of course, clear that a cable fixed at its two ends and uniformly loaded would take up this shape (see Fig 14 *b*). Similarly, a moment diagram produced by various concentrated loads is a series of straight lines, and an experiment shows that a string loaded with weights proportional to these loads with its ends secured will take up a shape exactly the same as that of the moment diagram when the weight of the string is negligible (see Fig. 14 *c*).

If, however, the weight of the string becomes comparable to the concentrated weights the moment diagram, as explained above, becomes a series of curves kinked at the points of application of the loads, corresponding exactly to the deflection that would take place in a loaded cable owing to the weight of the cable, which deflection becomes less as the weight of the cable becomes small proportional to the tension in it (see Fig. 14 *d*). This simile between the loaded cable and the moment diagram on a stiff structure is often very helpful in enabling the problems to be more easily visualized.

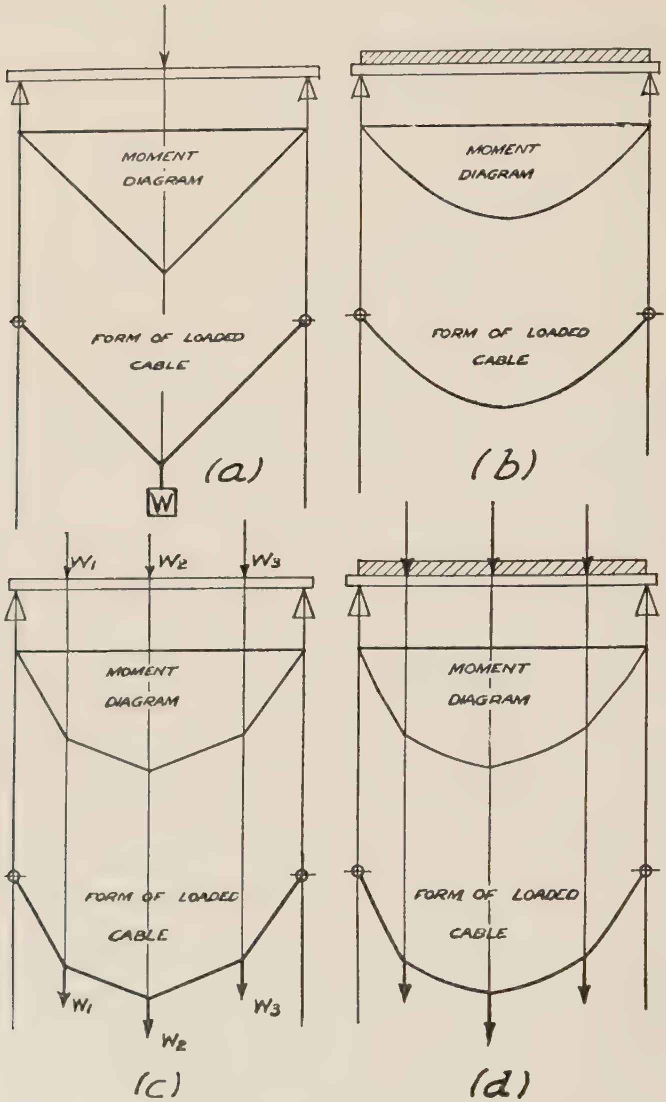

MOMENT DIAGRAM
FORM OF LOADED CABLE
W
(a)
MOMENT DIAGRAM
FORM OF LOADED CABLE
(b)
W_1
W_2
W_3
MOMENT DIAGRAM
FORM OF LOADED CABLE
W_1
W_2
W_3
(c)
MOMENT DIAGRAM
FORM OF LOADED CABLE
(d)

FIG. 14.

CHAPTER V

RESISTING MOMENTS

WE have now sufficiently studied bending-moment diagrams in a few simple cases to have formed some ideas on the subject and to enable us to pass on with profit to the next stage in our inquiry, which concerns itself with the resistance of beams or other stiff structures to such bending moments. For this purpose let us consider the simple cantilever (a beam fixed at one end and unsupported at the other) which is built into a wall at one end and carries a load W at the other.

If we consider the section AA, see Fig. 15 (a), at a distance l from the load W, it is clear that the bending moment at this section is

$$M = Wl.$$

The question that we now have to concern ourselves with is what happens in the beam at the section AA to enable it to resist this bending moment. We will assume that the beam in question is of the steel joist section, which consists essentially of two flanges separated by a thin web. To throw more light on the condition of affairs at the section AA, let us imagine the beam cut through at this section, when a little consideration will show that the effect of the load W would be to produce rotation as shown in Fig. 15 (b), the rotation being about the point 1 in the bottom flange.

To bring the beam back to its original shape and position it would clearly be necessary to apply a tension T to the top flange, and it will be clear that if we pull the top flange back by means of such a tension we can restore the beam to its original form and shape, showing that there was, in fact, such a tension acting in the top flange and performing this duty before we cut through the beam at this section.

Suppose now, on the contrary, we cut a V notch out of the beam from below, as shown in Fig. 15 (c); it is equally clear that the projecting portion of the beam will rotate about the point 2 at the upper flange unless we insert a block capable of resisting compression in the bottom flange, showing that the bottom flange, before the notch was cut, was already performing this duty and resisting compression.

We have therefore established the fact that a bending

moment in a beam is resisted by tension and compression forces acting in the flanges of the beam, and that in a cantilever the upper flange is in tension and the bottom flange is in compression. This tension and compression are indicated in Fig. 15 (*d*)

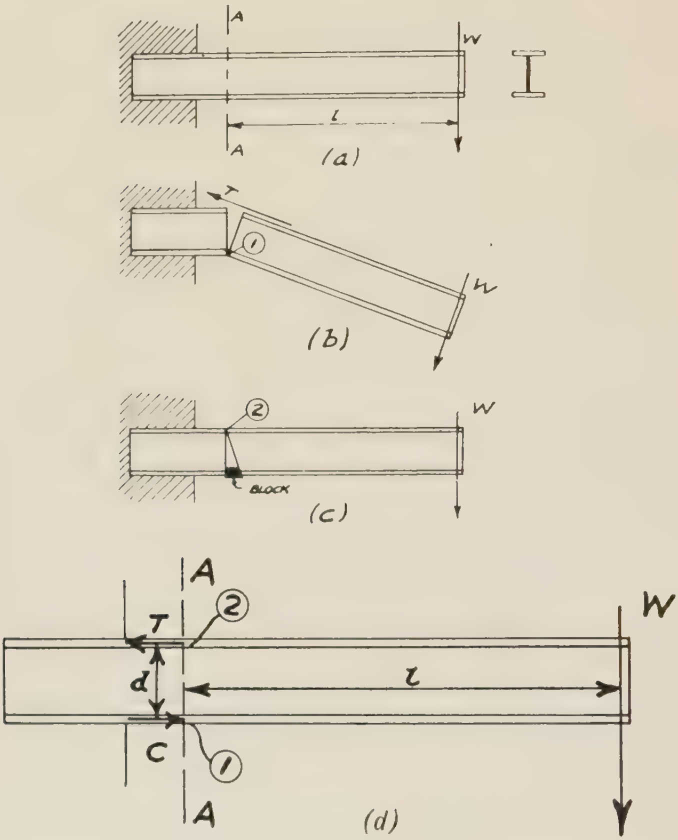


FIG. 15.

and may for the moment be taken as acting in the centres of the two flanges respectively and separated by a distance *d*.

It will be remembered that we have already shown that in a stable structure the moments about any point must balance. Let us now take moments about the point 1, this being where the compression flange intersects the section *AA* we are considering. We have here two forces producing moments, the

force W at a leverage l in a clockwise direction, which is resisted by the tension T at a leverage d in a counter-clockwise direction. The force C obviously produces no moment about the point 1, as it has no leverage. Therefore we may write

$$T \times d = W \times l,$$

or,

$$T = \frac{Wl}{d}.$$

In a similar way we can take moments about the point 2, when we find a clockwise moment, Wl, is equated and balanced by the counter-clockwise moment $C \times d$, so that

$$C = \frac{Wl}{d}.$$

The following facts emerge from this investigation :

(1) The total tension and total compression in the flanges of a beam are equal. This was, of course, deducible from first principles, since, as they are the only horizontal forces acting, as is clear from Fig. 15 (d), the structure would move laterally if one of these exceeded the other.

(2) The external bending moment is resisted by the moments of the internal forces, which is equal to the tension in one flange or the compression in the other flange multiplied by the distance between them. This moment is called the *resisting moment*, and we therefore see that in general the resisting moment

$$\begin{aligned} R &= T \times d \\ &= C \times d, \end{aligned}$$

d being the distance from the centre of one flange force to the centre of the other.

This simple relationship enables us to calculate the flange forces in any beam where we know the bending moments, which we have, of course, already discussed. In other words we may put in general—

The bending moment due to the external forces is equal to the resisting moment

$$\begin{aligned} M &= R \\ &= \mathrm{T} \times d \\ &= C \times d. \end{aligned}$$

We have already shown how M can be calculated in most ordinary cases, and d will be known for any given beam, making it very easy to evaluate C or T. If in addition we know the area of the flange which resists this total tension or total compression,

it is easy, of course, to go a stage further and calculate what the stress in the material comprising the flanges must be. This really comprises the essential principles in the design of ordinary beams and girders, and holds good however large and complicated the structure may be. It is just as true of the Forth Bridge as it is of the simplest girder of insignificant proportions.

Example. A joist 12 in. deep has flanges 6 in. wide and 1 in. thick top and bottom. The joist carries a concentrated load at midspan on a span of 20 ft. Calculate what load it will carry in this way for the stress in the flanges not to exceed $7\frac{1}{2}$ tons per sq. in.

The bending moment is

$$\begin{aligned} M &= \frac{Wl}{4} \\ &= \frac{W \times 240 \text{ in.}}{4} \\ &= 60\ W \text{ inch units.} \end{aligned}$$

The safe tension or compression in the flanges is

$$\begin{aligned} T &= C \\ &= 6 \text{ sq. in.} \times 7\tfrac{1}{2} \text{ tons per sq. in.} \\ &= 45 \text{ tons.} \end{aligned}$$

The distance from the centre of one flange to the centre of the other is 11 in. Therefore the resisting moment is

$$\begin{aligned} R &= Td \\ &= 45 \text{ tons} \times 11 \text{ in.} \\ &= 495 \text{ inch tons.} \\ M &= R \\ \text{Therefore } 60\ W &= 495 \text{ inch tons.} \\ \text{Therefore } \quad W &= \frac{495}{60} \\ &= 8\tfrac{1}{4} \text{ tons.} \end{aligned}$$

In this example it will be noticed that the span l is expressed as 240 in., not as 20 ft., because it is more convenient to express the depth of the joist in terms of inches than of feet. It does not matter at all what units we work in, but it is, of course, of paramount importance that whatever units we select we should stick to them. In this case tons and inches are used throughout as the units of weight and of length respectively.

It is impossible to emphasize too strongly the necessity for

the utmost care on the part of the student in maintaining the same units throughout in calculation, as probably more serious errors have been caused through failure to do this than through any lack of refinement in theories underlying calculations of this character.

If A is a cross-sectional area of the flange and f the stress in it we can clearly write

$$T = A \times f,$$

so that the resisting moment may be written

$$\begin{aligned} R &= T \times d \\ &= A \times f \times d. \end{aligned}$$

In other words, the resisting moment for a beam of this type is the stress times the flange area times the depth from one flange to the other. This makes it quite clear that the strength of a beam having a definite sized area of flange is directly proportional to its depth, that for a beam of a given depth the strength is directly proportional to the flange area.

We have shown how to calculate the resistance moment of the steel joist type of section characterized by the bulk of the material being concentrated in two flanges and therefore nearly all acting at the same distance from the neutral axis.

We now have to consider other sections used in practice where this approximation cannot conveniently be made. Clearly one of the most important sections is the rectangular one, which we must now consider. Before we can do so we must study a few important points applicable to all beams.

Consider a beam initially straight and supported at its ends as shown in Fig. 16 (a). When this beam is loaded it deflects. This deflection may not be sufficient to be obvious to the naked eye, but if apparatus suitable for the purpose is used it indicates a definite deflection corresponding to the load. If the ends were originally at right angles to the beam, and therefore parallel, they will now slope as shown in Fig. 16 (a) and the point where they meet, somewhere above the beam, is the centre of curvature.

The top and bottom of the beam after bending form two circular segments, and obviously the segment of smaller radius will be shorter than the segment of greater radius, so we see that these two faces, initially of equal length, are now of different length, the upper face having shortened and the lower face having lengthened. This shortening and lengthening is indicated at the end of the beam in Fig. 16 (a). The shortening

of the upper face is the effect of the action of compression stresses in it, while the lengthening of the bottom face is the effect of tensile stresses, and these shortenings and lengthenings are quite inevitable and in accordance with the operation of Hooke's law, which we studied previously.

Between the top layer, which is shortened, and the bottom layer, which is lengthened, there is a layer which is neither shortened nor lengthened. This is called the neutral plane. In symmetrical homogeneous sections it lies on its centre line. In unsymmetrical sections it passes through the centre of gravity of the section. From what has been said and from the diagram of the end of the beam after bending, shown on Fig.16 (a), it is clear that the lengthening or shortening will be as shown on Fig. 16 (b), which is, in fact, a strain diagram, the strain being a maximum at the top and bottom, getting gradually less as the neutral axis is approached and being compressive strain or shortening above the neutral axis, and tensile strain or lengthening below the neutral axis. From this and from what has been previously said it is clear that *the strain is proportional to the distance from the neutral axis.* In Fig. 16 (b) the strain at a layer half-way between the neutral axis and the top is indicated and is clearly equal to half the strain at the top.

From Hooke's law, with which we made acquaintance earlier, we saw that the stress is proportional to the strain. Therefore, as the strain is proportional to the distance from the neutral axis, it also follows that *the stress is proportional to the distance from the neutral axis.* It follows that the distribution of stress over the section is as shown in Fig. 16 (c), being exactly similar to the strain diagram, the maximum stress (generally denoted by f) being at the top and bottom of the beam.

The portion of the beam above the neutral axis is the compression flange, and the portion below the neutral axis is the tension flange, and if we can calculate the total compression (or the total tension) and multiply this by the distance from one to the other we shall have calculated the resistance moment as before.

The total compression is clearly the area in compression multiplied by its average stress. The area in compression is

$$b \times \frac{d}{2}$$

and the average stress (being the average between f at the top

and zero at the bottom of the compression area) is clearly $\frac{f}{2}$, so that the total compression is

$$C = b \times \frac{d}{2} \times \frac{f}{2}.$$

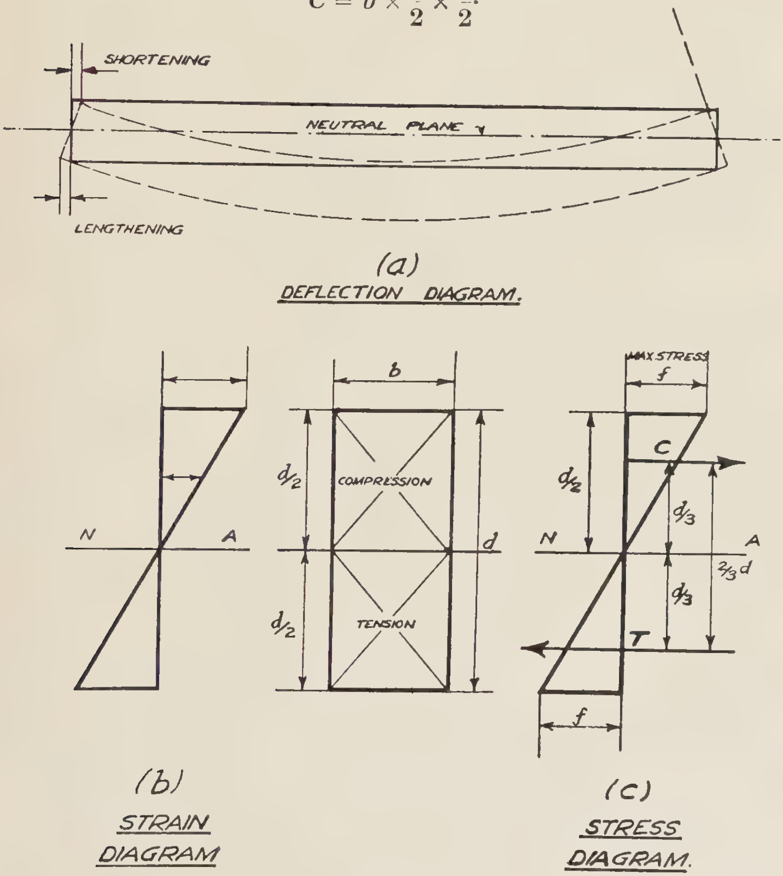


FIG. 16.

It only remains now to calculate the distance from the centre of compression to the centre of tension. If the stress in the compression area were uniform the centre of compression would lie centrally in the compression area. But the stress in the compression area varies from nothing at the neutral axis to a maximum at the top in a triangular distribution. The centre of compression in a case like this lies at the centre of gravity of the stress diagram, and the centre of gravity of a triangle is at

a distance of two-thirds the height from the apex. In Fig. 16 (*c*) the height of the triangle is $\frac{d}{2}$, and therefore the centre of gravity of the triangle is at a distance of two-thirds of this, viz. $\frac{d}{3}$ from the neutral axis. Similarly, the total tension T is at a distance of $\frac{d}{3}$ from the neutral axis, so that the distance from the centre of compression to the centre of tension is two-thirds of d. The resistance moment is therefore given by

$$R = \text{total compression} \times \text{distance}$$

from total compression to total tension

$$= b \times \frac{d}{2} \times \frac{f}{2} \times \frac{2}{3} d$$

$$= \frac{f.b.d^2}{6}.$$

It is interesting to compare this with the corresponding expression for the steel-joist section :

$$R = f \times Ad.$$

In both cases the resistance moment is made up of two expressions, the first being the stress and the second being an expression depending solely on the dimensions of the specimen and having the value $A \times d$ for the steel-joist section and $\frac{bd^2}{6}$ for the rectangular section. These expressions are called the *section modulus.*

The section modulus may be defined as the quantity which, multiplied by the stress, gives the resistance moment of the beam. We have seen that it may be calculated by adding together the areas of the flanges multiplied by their distance from the neutral axis. The section modulus is generally denoted by the letter Z, so that we may write generally

$$R = f \times Z.$$

The resistance moment therefore consists of two quantities, the first of which is the stress and depends on the strength of the material used, while the second quantity depends only on the dimensions of the section.

Several important points result from what we have done. In the first place the expression for the section modulus of the steel joist (area of flange times depth) shows that a deep beam is proportionately stronger than a shallow beam when both

have the same area of flange, and therefore the same weight. From this it follows, of course, that deep beams are more economical than shallow ones, and this is invariably true until the web (the material connecting the two flanges) becomes a dominating consideration.

Secondly, we notice from the section modulus of the rectangular beam $\left(\frac{bd^2}{6}\right)$ that the depth is more important than the breadth. For example, a 9×3 plank on edge has a strength proportional to

$$\frac{3 \times 9^2}{6} = 40\tfrac{1}{2},$$

while the same beam on the flat has a strength proportional to

$$\frac{9 \times 3^2}{6} = 13\tfrac{1}{2},$$

so that a beam on edge is three times as strong as the same beam on the flat when the ratio of breadth to depth is 3. Depth is not only important as giving strength in the most economical manner, but it also makes for stiff structures, that is, structures with a minimum of deflection. For rectangular sections it can be shown that although the strength varies with the square of the depth, the stiffness varies with the cube of the depth, showing what an important part the depth plays in the stiffness of a structure.

In steel joists, which are rolled from solid ingots of white-hot metal by machinery of enormous cost, certain standard sections are adopted, and designers must make use of these sections and get the strength they require from a combination of these sections with or without other sections or plates riveted to them. It would take an order of something quite outside the ordinary to justify putting down fresh rolls for a section other than the standard one. For these standard sections the section modulus has been worked out once and for all, and is given in the lists issued by the British Standards Association, for example, and by the various steel constructional firms. If the section modulus corresponding to any particular joist is multiplied by the safe stress (say, $7\frac{1}{2}$ tons per sq. in.) the result is the resistance moment of that joist expressed in inch tons. The student should check some of these figures taken from the steelworks list against the figure obtained by adopting the simple formula 'Area of flange times distance between flanges', and he will see that the approximation is very close, the differences arising being due to the somewhat complicated shape of

the actual steel joists necessitated by the practical requirements of rolling and of reducing the shrinkage stresses on cooling to a minimum.

It is important to realize that the law that the stress in a beam varies directly as the distance from the neutral axis, although only derived and used so far in connexion with beams of rectangular section in what has preceded, is of quite general application, as may be seen by noting the argument on which it was deduced, and therefore applies equally to beams of steel-joist section.

The reason this did not affect our formula for the resistance moment of steel joists was because the thickness of the flange was small compared to the distance from the neutral axis, and therefore the variation of stress between the two edges of a flange was small.

When a flange is greatly thickened by riveting several additional plates to it, allowance must, however, be made. We may then work to the average stress, and should select this stress somewhat below the maximum permissible, so that the maximum, which, of course, occurs at the extreme edge, shall not be excessive. This may be done as in the example which follows.

We also need to make allowance for the holes drilled through the flanges for the rivets. Even if the rivets fill the holes completely—and therefore allow of compressive stresses across them—clearly no tension stress can occur across the holes in the tension flange, and hence the area of such holes is deducted from the area of flange when calculating the resistance moment.

Both these little refinements are exemplified in the following: Fig. 17 shows a built-up beam consisting of an 18 in. by 7 in. steel joist, to which are riveted four 12 in. by $\frac{3}{4}$ in. plates to each flange, two rows of $\frac{3}{4}$ in. rivets being used on each flange, the area of flange of the joist being 7 in. by 1 in.

The strength of this would be calculated as follows:

	Area of flange sq. in.
Four plates, 12 in. by $\frac{3}{4}$ in.	36
Joist flange, 7 in. by 1 in. . . .	7
Gross area	43
Deduct for rivets:	
Twice 4 in. by $\frac{3}{4}$ in.	6
Net area	37

The centre of gravity of the flange is $10\frac{1}{4}$ in. from the neutral axis, while the extreme edge is 12 in.

Hence, if the stress of $7\frac{1}{2}$ tons per sq. in. is not to be exceeded at the extreme edge, the average stress (measured at the centre of gravity) must be

$$7\tfrac{1}{2} \times \frac{10\frac{1}{4}}{12} = 6{\cdot}4 \text{ tons per sq. in.}$$

Hence the resistance moment is average stress $\times$ net area $\times$ distance between flanges.

$$R = \left(7\tfrac{1}{2} \times \frac{10\frac{1}{4}}{12}\right) \times (37 \text{ in.}^2) \times (2 \times 10\tfrac{1}{4} \text{ in.})$$

$$= 4{,}860 \text{ inch tons.}$$

The section modulus is the resistance moment divided by the *maximum* stress ($7\frac{1}{2}$ tons per sq. in.).

$$Z = \left(\frac{10\frac{1}{4}}{12}\right) \times (37) \times (2 \times 10\tfrac{1}{4})$$

$$= 648 \text{ inch}^3 \text{ units.}$$

It will be noticed from Fig. 17 that the inner faces of the joist are tapered (to facilitate rolling). This creates no special difficulty when rivets are used, as these accommodate themselves to the slope while they are 'closed' hot. But bolts require *tapered washers* to prevent the boltheads bearing on one edge only.

Reference to Figs. 10 to 14 will show that bending moments are generally greatest at midspan (or thereabouts) and die away to zero at the ends. Clearly the resistance moment may, without loss of safety, be similarly reduced, and this is easily done (and economy of material effected thereby) if the plates are stopped off as we approach the ends of the beam.

This is more fully explained by reference to Fig. 18, which shows the moment diagram of a beam with uniformly distributed load as a parabola. We will assume that the beam shown in Fig. 17 is required to resist this moment.

The maximum moment at midspan, then, requires a net flange area of 37 sq. in., made up as follows :

	sq. in.
Flange of joist (net)	$5\frac{1}{2}$
Four plates ($10\frac{1}{2} \times \frac{3}{4}$ net $= 7\frac{7}{8}$ sq. in. net) .	$31\frac{1}{2}$
	37

We may find a scale so that the whole depth of the moment diagram gives 37 sq. in. to this scale.

Then this distance can be subdivided to this scale to indicate the various plates, as shown, the depth allotted to each being proportional to its sectional area.

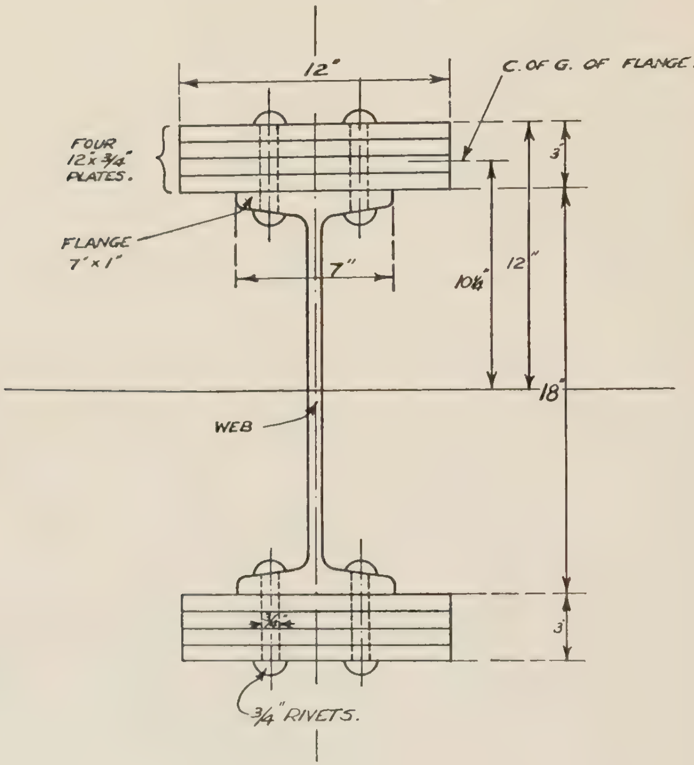


Fig. 17.

It is clear that while the whole area is required at *a* the moment at *b* (where the moment diagram crosses the horizontal line indicating the top of the extreme plate) has been so reduced that this plate is no longer required. Similarly, the remaining plates may be stopped at *c*, *d*, and *e* respectively, the joist flange alone requiring to go the whole length to *f*. The upper diagram (being a side elevation of the beam) shows how these points are used in determining the length of plates. Actually in practice we usually run the plates 3 to 6 in. beyond

the points so determined, partly because the plate does not act till the first of the rivets is reached, and partly as a provision against slight variation from the ideal moment diagram.

It is clear that the shaded area represents the metal saved to the same scale, as the area *a p q n* represents the weight of half the beam. In the extreme case of many plates and single-

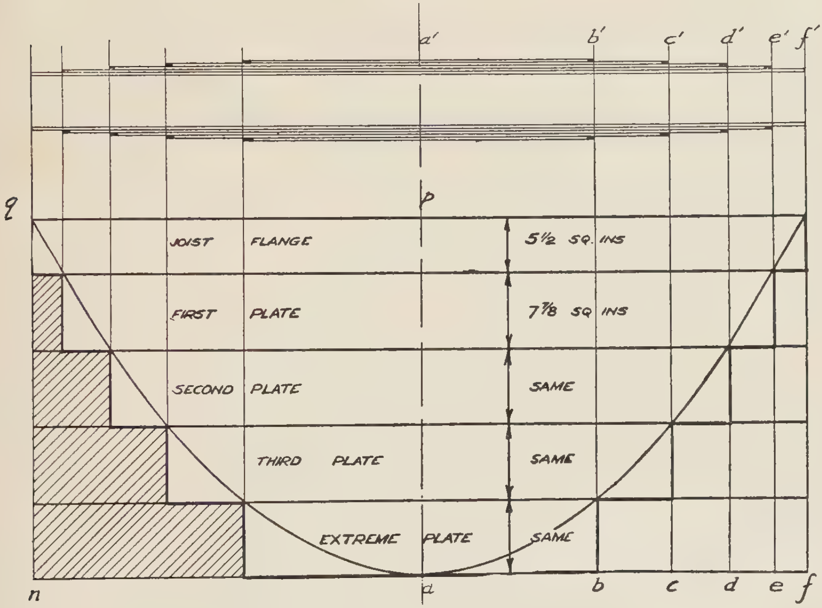
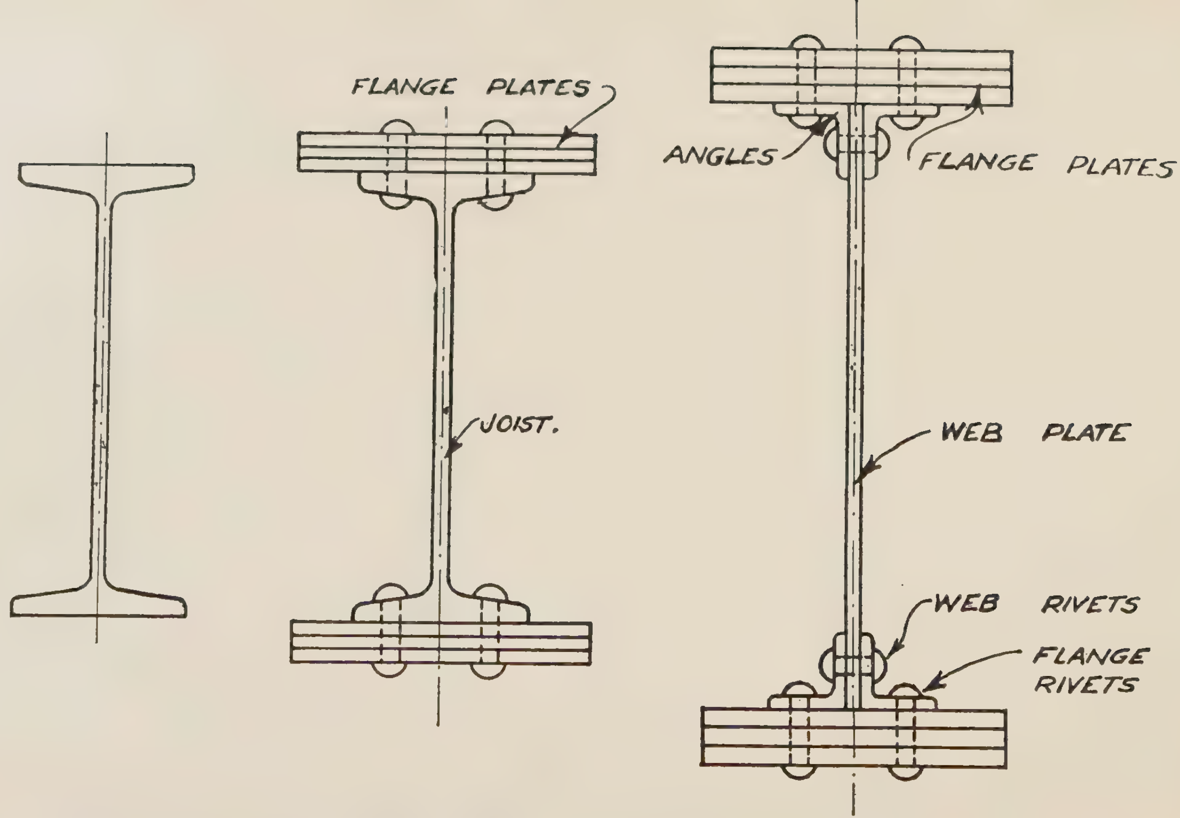


FIG. 18.

point loading, this saving may approach 50 per cent., and with uniform loading may approach 33⅓ per cent. Actually in practice more usual figures are 30 and 20 per cent. respectively. Clearly this is sufficiently important.

Fig. 19 shows various types of steel beams. The standard joist is always used if possible, selected so as to give the requisite resistance moment. If the largest obtainable standard joist (generally 24 in. deep) is not adequate we may use a standard joist plated. Sometimes the appropriate steel joist is too deep. Then we may use several smaller ones side by side, or a smaller one plated. Sometimes even the largest joist plated is not strong enough. Then we may use the *plate girder*, which has a deep plate for the web and is connected to the



STEEL JOIST PLATED BEAM PLATE GIRDER

Fig. 19.

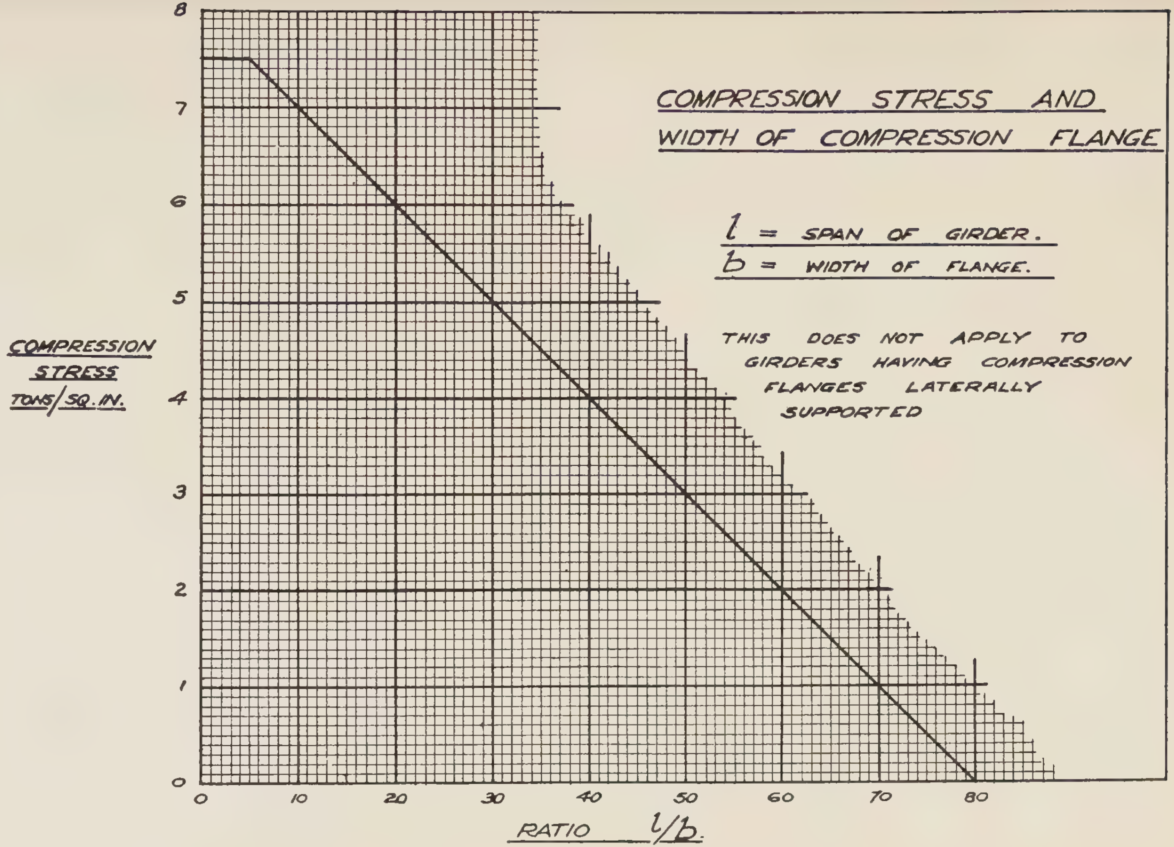

COMPRESSION STRESS AND
WIDTH OF COMPRESSION FLANGE
l = SPAN OF GIRDER.
b = WIDTH OF FLANGE.
THIS DOES NOT APPLY TO GIRDERS HAVING COMPRESSION FLANGES LATERALLY SUPPORTED
COMPRESSION STRESS TONS/SQ.IN.
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
RATIO l/b.

FIG. 20.

flanges by angles. Such a girder requires web rivets as well as flange rivets for connecting to the angles in either case.

Occasionally an open steel lattice is substituted for the continuous web plate. This constitutes a *lattice girder.*

We have now considered the technique underlying the calculation of the weight of flanges necessary to a girder. There is only one word of warning in connexion with this before the student can apply what we have done to any practical case. This relates to the question of buckling of the compression flange. We shall see in greater detail later, when we come to consider stanchions—and it is indeed a matter of everyday experience observed by every one—that beyond a certain limiting length, members in compression of great length are weaker than the same compression members shorter. A walking-stick, for example, will seldom carry the whole weight of a man on its full length, but a short section of walking-stick, say, 2 in. long, would carry many men. The upper flange of a girder is, as we have seen, simply a compression member, and the same considerations apply. It is fortunately prevented from buckling upwards by being connected throughout its length to the web, which is very stiff in this direction. But there is nothing to prevent its buckling horizontally. It is therefore necessary either to give the compression flange lateral support or, alternatively, to limit the stress in relation to its slenderness.

Most girders in the floors of buildings have their compression flange contained in a floor of reinforced concrete, which gives it the lateral stiffness required. In such cases the slenderness of the compression flange requires no reduction of stress.

But occasionally we come across examples of girders where no such side support is given, and such cases require very careful consideration. A bridge consisting of two girders, with the deck carried between them on the lower flange, would be an example of such a case.

The curve in Fig. 20 shows how such cases can be met by a reduction of stress in the compression flange, the curve showing the appropriate stress which is safe corresponding to various ratios of span to width of compression flange. From what has been said it will be clear that this curve is not to be used where the compression flange is already supported laterally.

The L.C.C. Regulations (General Powers Act, 1909) dealt with it otherwise. They require, in Section VI : ‘ Every girder

shall be secured against buckling whenever the length of the girder exceeds thirty times the width of the compression flange.' This means, presumably, that the full stress may be used right up to this limit, and beyond that the use of the girder is prohibited altogether. It would appear more rational to require some increase in strength before this limit is reached and to allow girders beyond this limit to be used, provided a correspondingly greater increase in strength is given, and the curve has this effect.

It will be seen that a girder with a 6-in. upper flange unsupported laterally would be stressed to $5\frac{1}{2}$ tons per sq. in. if it were 12 ft. 6 in. long, $4\frac{1}{2}$ tons per sq. in. if it were 17 ft. 6 in. long, and so on in proportion, the absolute limit of length for a 6-in. flange being 40 ft., and in any length beyond 20 ft. involves great loss of economy.

CHAPTER VI

SHEAR AND WEB STRESSES

SO far we have not dealt with the webs of beams or girders, but they form a very necessary part and are just as essential as the flanges.

In Fig. 21 is shown a cantilever in which a portion of the beam has been replaced by means of links *a b* and *c d*, the former being, of course, in tension and the latter in compression. If no diagonal members are introduced the action of the load will be to lower the right-hand portion of the cantilever in a manner as shown in Fig. 21 (*b*). This shows that while the flanges *a b* and *c d* respectively are necessary they are not sufficient to ensure the stability of the girder. If, however, the point *a* is attached to the point *d* by a diagonal member, or the point *b* to the point *c* by another diagonal member, then the lowering of the girder, as shown in Fig. 21 (*b*), will be prevented if these diagonals are made strong enough.

It will be seen that the length *a d* in Fig. 21 (*a*) has been increased by the dropping of the girder in Fig. 21 (*b*), while the length of *b c* in Fig. 21 (*a*) has been reduced in Fig. 21 (*b*) by the dropping of the girder.

Remembering that the stress which tends to lengthen a member is tension and the stress which tends to shorten it is compression, it is obvious that the stresses in the web of a girder consist of tension stresses in one diagonal direction and compression stresses in the other. The web system is therefore sometimes referred to as the diagonal system. If these diagonals take the form of isolated members, such as *a d* and *b c*, then the girder is referred to as a lattice girder ; but they may be replaced by a web plate, in which case this plate is subject to diagonal stresses, already explained. It is therefore clear that the webs of girders are subject to diagonal tension in one direction and diagonal compression approximately at right angles to the tension.

Although it can be shown that the stresses in a web can be explained simply by tension and compression they are frequently referred to as shear stresses, and this conception is convenient from some points of view.

If a piece of material, as in Fig. 22 (*b*), is put into a pair of

shears it will be cut by the left-hand portion being forced up and the right-hand portion down. This is referred to as a shearing action, and plates are cut in this way by large machines in a steelworks, referred to as shearing machines. If a girder section such as that shown in Fig. 22 (a) is considered it will be clear that the girder to the left of the section tends to be pushed upwards by the reaction R relatively to the right-hand section, which tends to be pushed down by the force W. The material in the section has to be strong enough to resist this tendency, and hence the web, by analogy, is said to be in a state of shear, and the stresses in it are referred to as shear stresses.

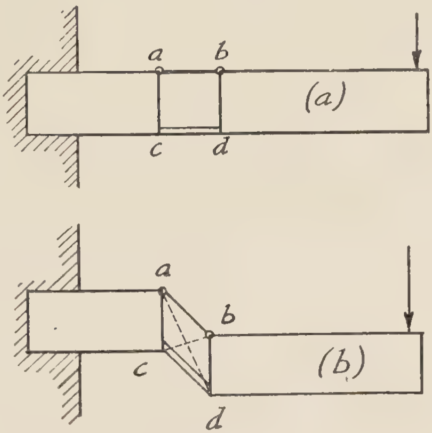


FIG. 21.

It can be shown by experimental work that the resistance to shear varies directly with the cross-sectional area of the material subject to shear. If we divide the total shear across a section by the number of square inches of web resisting this shear the product is known as the shear stress. Thus, if a web is 10 in. deep and $\frac{1}{2}$ in. thick the area of the web is 5 sq. in.; and if the shear across it is 25 tons then the shear stress would be

$$s=\frac{S}{A}=\frac{25 \text{ tons}}{5 \text{ sq. inches}}=5 \text{ tons per sq. inch.}$$

Let us now consider the web shown in Fig. 23 (a).

The total shear S is resisted by the total web. Therefore 1 in. of web in height would resist a portion of it. If we imagine for simplicity that the web is 1 in. thick then the portion resisted in 1 in. height would be the shear per sq. inch. This is arrived at by dividing the total shear by the area of the web.

Let us consider a little square of material, having length of side equal to 1 in. The two vertical faces are subject to little shearing forces, equal to s_1 s_2, this being the shear stress in the web. If these were the only two forces acting on the square,

it is obvious that they would set it into rotation in a clockwise direction, which, of course, we know from experience does not actually occur. It is therefore obvious that there are other forces acting on this little unit of material to prevent it from rotating, and the only other forces which could have this effect are the horizontal forces s_3 and s_4 indicated in the diagram. To prevent rotation not only must they be in the direction shown, but they must be equal in magnitude to s_1 and s_2. These forces s_3 and s_4 obviously indicate a state of shear stress in a horizontal plane, and we therefore have the following important law :

Any material subject to a vertical shear stress is also subject to an equal horizontal shear stress. The existence of this horizontal shear stress will be made clear later.

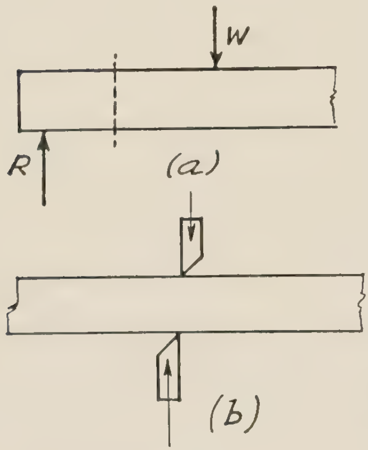


FIG. 22.

These four unit forces acting on our small square of material can now be combined. We may first combine s_3 and s_2 to produce a diagonal resultant whose value will be $\sqrt{2}\, s$, as shown in Fig. 23 (*b*); similarly, s_1 and s_4 can be combined, and the resultant will be a diagonal having the same value as before and opposite to it in direction. These two resultants clearly serve to put the material inside the square in a state of compression, and they exert compression on an area of material at right angles to the force having a width equal to the length of the diagonal, which is $\sqrt{2}$. As we took the web of unit thickness the area on which the forces act is therefore

$$\sqrt{2} \text{ sq. in.}$$

The compression stress is therefore

$$c = \frac{\sqrt{2}\, s}{\sqrt{2}}$$
$$= s.$$

Our elemental forces can be combined in another way. We

can combine s_1 with s_3 to form a resultant in a diagonal direction, having the magnitude of

$$\sqrt{2}\, s \text{ (see Fig. 23 } (c)),$$

and similarly s_2 and s_4 can be combined to form a resultant in the same line but opposite to the previous one. These resul-

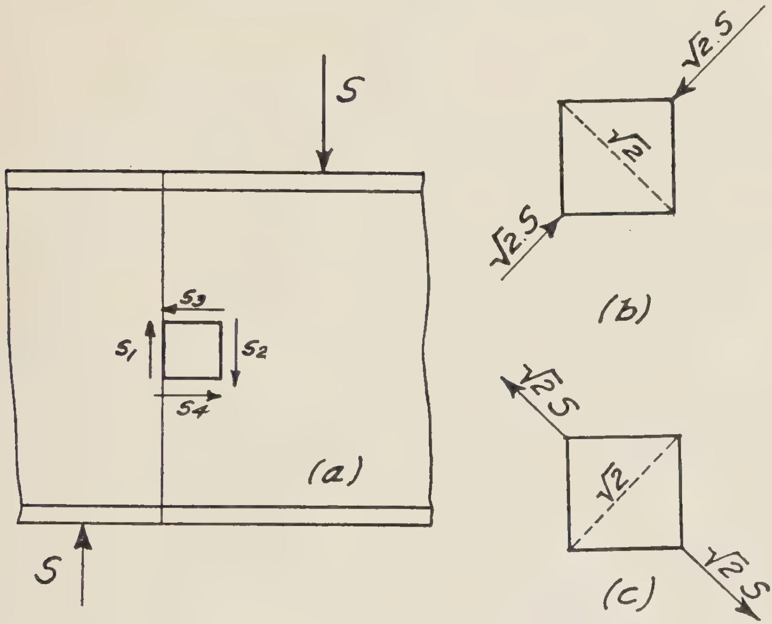


FIG. 23.

tants obviously produce tension across the diagonal of the square whose area is $\sqrt{2}$ in the one direction and unity in the other, and therefore has an area of $\sqrt{2}$ sq. in., so that the tension stress on the other diagonal is

$$t = \frac{\sqrt{2}\, s}{\sqrt{2}}$$

$$= s.$$

This shows that *a vertical shear stress necessitates an equal horizontal shear stress, and that these can be combined to produce diagonal compression at 45° and diagonal tension at right angles to it, the tension and compression stresses being equal to the shear stresses.* This is an extremely important result and very much

facilitates a proper understanding of what goes on in the web of a girder.

With webs such as are common in steel girders, where the thickness is generally small compared with the depth, it is pretty clear that failure will occur by the steel buckling under the compression stress at 45°, and this is, in fact, how such webs fail.

In a reinforced concrete beam, however, owing to the fact that concrete is much weaker in tension than in compression, coupled with the fact that the ordinary proportions of a concrete beam involve so great a thickness as compared with the depth as would preclude any idea of buckling, the beam will fail by tension on a diagonal plane producing diagonal cracks, and this is, in fact, how a concrete beam fails in shear.

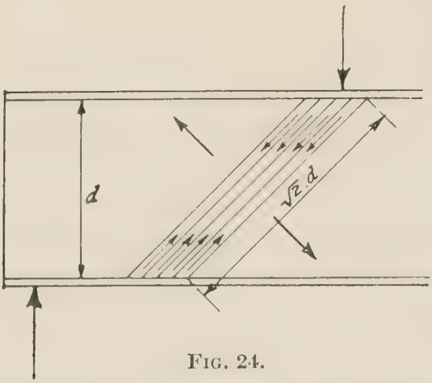


Fig. 24.

It will be remembered that we were discussing the stresses in the webs of plate girders, and we showed that if the shear across a section divided by the area of the web was called the vertical shear stress, there then existed an equal horizontal shear stress, and that these two stresses could be combined to produce diagonal compression at 45°, and diagonal tension at right angles to it, *the intensity of the compression and tension being equal to that of the shear stress.* In other words, if we want to limit the compression stress in a web to a safe value of, for example, $5\frac{1}{2}$ tons per square inch, the relationship just quoted shows that to do this it is only necessary to limit the vertical shear stress to the same value. This generally results in a very simple calculation.

Just as we saw that the compression flange of a girder was much more liable to buckle when it was long, as compared with its width, and therefore necessitated a lower stress with an increasing slenderness ratio, so the webs of girders in compression diagonally are liable to buckle when the ratio of length to thickness of these diagonal strips increases.

In Fig. 24 the web is considered as a series of diagonal strips,

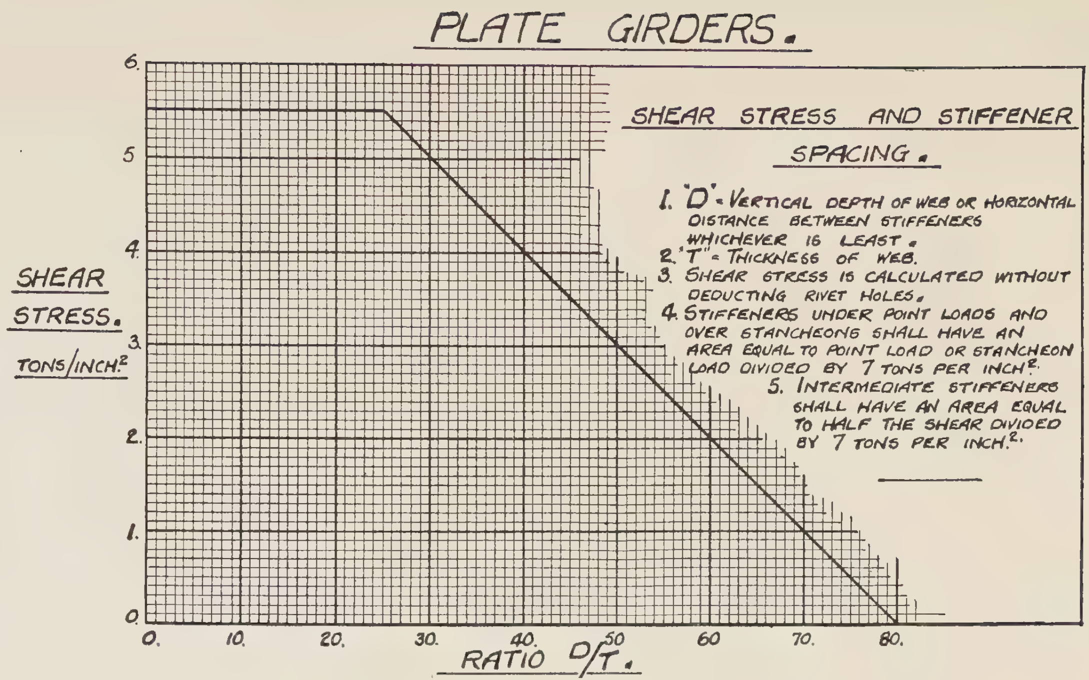


PLATE GIRDERS.
SHEAR STRESS AND STIFFENER SPACING.
1. "D" = VERTICAL DEPTH OF WEB OR HORIZONTAL DISTANCE BETWEEN STIFFENERS WHICHEVER IS LEAST.
2. "T" = THICKNESS OF WEB.
3. SHEAR STRESS IS CALCULATED WITHOUT DEDUCTING RIVET HOLES.
4. STIFFENERS UNDER POINT LOADS AND OVER STANCHEONS SHALL HAVE AN AREA EQUAL TO POINT LOAD OR STANCHEON LOAD DIVIDED BY 7 TONS PER INCH^2.
5. INTERMEDIATE STIFFENERS SHALL HAVE AN AREA EQUAL TO HALF THE SHEAR DIVIDED BY 7 TONS PER INCH^2.
SHEAR STRESS. TONS/INCH^2
6.
5
4.
3.
2.
1.
0
0.
10.
20.
30.
40.
50
60
70.
80.
RATIO D/T.

Fig. 25.

each subject to a definite compression stress, and it will be clear that if d is the depth of the web the length of these strips will be $\sqrt{2}\,d$. There is therefore a definite ratio between the length of the strip and the depth of the web, so that if we express the allowable stress in the web in terms of the ratio

$$\frac{\text{depth of web}}{\text{thickness of web}}$$

this will be proportional to the slenderness ratio of the diagonal strips subject to buckling.

A curve is given in Fig. 25 showing the stress adopted in the author's practice for various values of this ratio. In preparing a diagram of this kind it is necessary to take into account the fact that the diagonal tensions at right angles to the compressions do to some extent reduce the tendency to buckle.

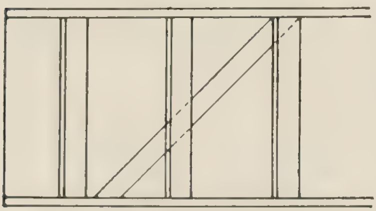


FIG. 26.

Reference to Fig. 26 will show that the introduction of stiffeners to the web at a spacing less than the depth of the web has the effect of reducing the buckling length of these diagonal strips just the same as if the depth of the web had been reduced to the distance between the stiffeners. Our curve in Fig. 25 can therefore be used to give the allowable stress in terms of the depth of the web where no stiffeners are used, or in terms of the spacing of the stiffeners where these are adopted. It will be clear that stiffeners further apart than the depth of the web do not reduce the buckling tendency appreciably, though they may serve other important functions.

As an example, in the design of a web of a girder to this curve, take the case of a girder having a depth of web of 30 in. (measuring inside the flanges) and a shear across the section of 60 tons. If we make the web $\frac{1}{2}$ in. thick the area of web would be

$$A = 30 \text{ in.} + \tfrac{1}{2} \text{ in.}$$
$$= 15 \text{ sq. in.}$$

The shear stress would therefore be

$$s = \frac{60 \text{ tons}}{15 \text{ sq. in.}}$$
$$= 4 \text{ tons per sq. in.}$$

The ratio of depth to thickness is

$$\frac{d}{t} = \frac{30 \text{ in.}}{\frac{1}{2} \text{ in.}}$$
$$= 60.$$

Our curve shows that for this ratio a stress of only 2 tons per sq. in. is permissible, so that the unstiffened web would not be satisfactory for 4 tons per sq. in.

We notice, however, from our diagram that 4 tons per sq. in. is quite satisfactory with a ratio of

$$\frac{d}{t} = 40,$$

which corresponds to

$$d = 40 \times \tfrac{1}{2} \text{ in.}$$
$$= 20 \text{ in.}$$

While we cannot reduce the depth of web to this value we can as a rule insert stiffeners at this spacing. This, therefore, is one solution of the problem, viz. a web $\frac{1}{2}$ in. thick with stiffeners 20 in. apart. There are, however, other solutions of the problem.

Suppose, for example, we try a $\frac{3}{8}$-in. web. The area of web would be

$$A = 30 \text{ in.} \times \tfrac{3}{8} \text{ in.}$$
$$= 11\tfrac{1}{4} \text{ sq. in.}$$

The shear stress would be

$$s = \frac{60 \text{ tons}}{11\frac{1}{4} \text{ sq. in.}}$$
$$= 5\tfrac{1}{3} \text{ tons per sq. in.}$$

Our diagram indicates that this stress corresponds to an allowable slenderness ratio of

$$\frac{d}{t} = 26\tfrac{2}{3},$$

so that the necessary spacing of stiffeners for this thickness of web would be

$$d = 26\tfrac{2}{3} \times \tfrac{3}{8} \text{ in.}$$
$$= 10 \text{ in.}$$

On the other hand we may try a thicker web, $\frac{5}{8}$ in., for example. The area of web would be

$$A = 30 \text{ in.} \times \tfrac{5}{8} \text{ in.}$$
$$= 18\tfrac{3}{4} \text{ sq. in.}$$

The shear stress would be

$$s = \frac{60 \text{ tons}}{18\frac{3}{4} \text{ sq. in.}}$$
$$= 3{\cdot}2 \text{ tons per sq. in.}$$

Therefore the allowable slenderness ratio would be

$$\frac{d}{t} = 48. \quad \text{(See curve.)}$$

Whence the allowable d is

$$d = 48 \times \tfrac{5}{8} \text{ in.}$$
$$= 30 \text{ in.}$$

We therefore have three alternative practical solutions, and the one we decide to adopt would depend on questions of cost and convenience. The three solutions are indicated in Fig. 27. The diagram gives a rule for the size of stiffener necessary. The stiffeners generally take the form of two steel angles back to back, riveted together with the web plate between them, the rivets generally being $\frac{3}{4}$ in. diameter at 6-in. centres for work of average magnitude.

From what has been said already it is clear that the function of the stiffeners is to prevent the two flanges from crushing together in a diagonal direction. It is therefore important that the stiffeners should extend right up to the flanges and be machined or forged to touch these continuously so that the construction shown in Fig. 28 (*a*) would be good, while that in Fig. 28 (*b*) would be bad. As an alternative to (*a*) the stiffeners may be joggled as shown in (*c*), but still require to be machined or forged at the ends. This arrangement is generally more expensive than (*a*). On a big plate girder having wide flanges it is good practice to bend some of the stiffeners as shown in (*d*), because this makes a much stiffer connexion between the web and the flanges. Quite a good arrangement is to alternate (*a*) or (*c*) with (*d*).

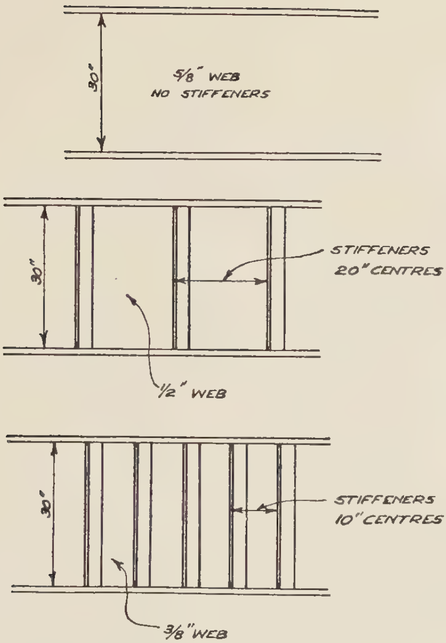


FIG. 27. Alternative Solutions for a Web to resist 60 Tons Shear.

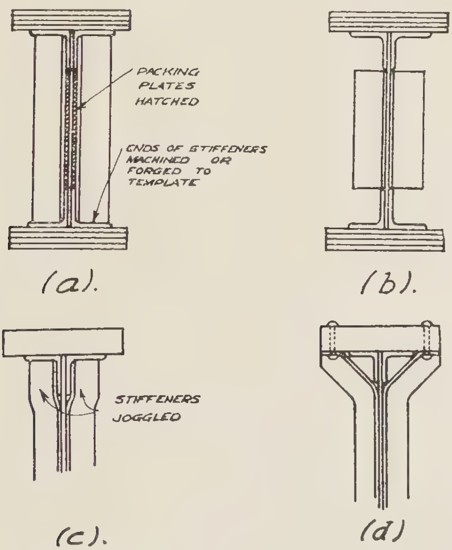


FIG. 28. Types of Stiffener Connexions.

CHAPTER VII

STANCHIONS

IT has already been explained, in connexion with the compression flange of girders, that when these are long in relation to their breadth they tend to buckle or bend, and if we are to design for a constant factor of safety it becomes necessary to adopt a reduced working stress as the slenderness ratio gets greater. The exact extent to which the stress has to be reduced with a given slenderness ratio is a matter requiring some considerable judgement and on which there is considerable confusion.

A mathematician of the name of Euler made some experiments on long struts and at the same time analysed the problem mathematically, and derived formulae which agreed substantially with experimental values. These experiments and analyses both applied to very long, slender struts where the direct stress (i.e. the load divided by the area) is very small compared with the stress due to bending when the strut begins to buckle. He found a marked difference according to whether the ends were fixed in direction (for example, by being rigidly clamped to the support), or whether the ends were free to rotate (as, for example, when they are supported by a spherical bearing, or when the end is hinged in trunnions, or even simply rounded).

When the ends are fixed the curvature in bending is necessarily a double one (see Fig. 29), whereas when they are hinged the curvature is a single one. Therefore a stanchion with fixed ends deflects less for a given curvature than a stanchion with hinged ends, and would therefore have a higher allowable stress for a given slenderness ratio.

As applied to practical conditions in building Euler's results are of limited applicability, because we do not use stanchions of the high slenderness ratios to which his formulae were applicable. On the contrary, a practical stanchion is generally used under conditions in which the direct stress is between 3 and 6 tons per sq. in. There is another difficulty with these formulae, and that is, that though in his experiments the conditions of fixed ends or of hinged ends could be approximated to, it is the exception for a practical stanchion in a building to

approximate to either. A practical stanchion generally has one end secured to a base plate of which we can only say that it is more nearly fixed than hinged. At the upper floors the stanchions are generally bolted or riveted rigidly to girders which are themselves subject to some deflection when they are loaded. If the girders are much stiffer than the stanchions, which may be the case near the top of the building, the stanchions may approximate to the condition of fixed ends, noting, however, that the ends, though fixed, are not fixed quite vertically owing to the deflection of the girders. The lower lengths of a stanchion in a high building are, however, frequently stiffer than the beams or girders to which they are connected, and in this case the discrepancy from fixed ends is greater.

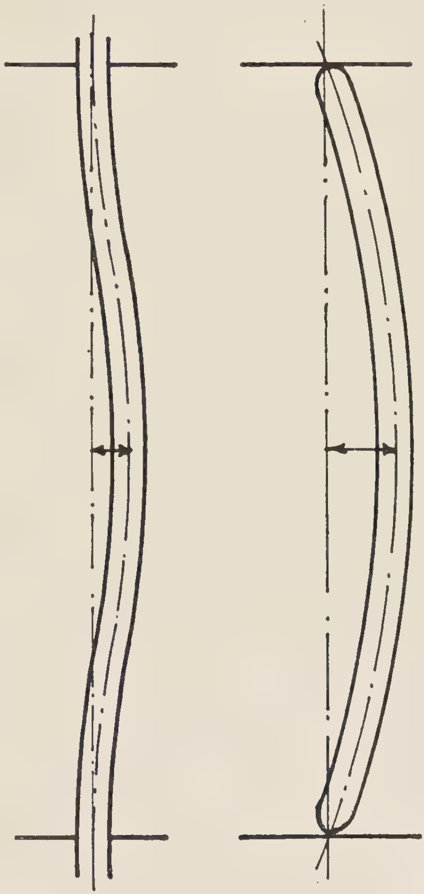


FIG. 29.

The L.C.C. regulations for stanchions are indicated on Fig. 30 by the full lines, and it will be seen that they follow Euler's treatment in regard to distinguishing between fixed and hinged ends, but not as regards the form of the curve or the stresses. It has already been explained that this conception of fixed or hinged ends has little meaning in practical work.

It may perhaps be permitted to criticize the L.C.C. curves without any intended disrespect to their originators, as we have perhaps a clearer knowledge of the subject than was the

case in 1909. The curves appear to be faulty in conception, not only because of the ambiguity as regards fixed and hinged ends in practice, but also for the following reasons.

With very short stanchions the question of buckling does not arise, and therefore there would be no difference in safe stress whether the ends were fixed or not. There is therefore no justification for two of the curves starting with a safe stress of 4·5 and 5·5 for short stanchions where buckling cannot arise. Obviously, for such short stanchions the full compression stress could be allowed. Secondly, it is clear that if $7\frac{1}{2}$ tons is permissible for the compression in the flange of a girder it ought to be equally permissible for a short stanchion where there is no risk of buckling.

A study of the actual experimental values of columns subject to buckling shows that the shape of the curve at high slenderness ratios tends to flatten out just where the L.C.C. curves become steeper, showing that the L.C.C. curves would have been not only simpler, but also more accurate, if the straight portion for the upper length had been allowed to run through straight instead of being bent down as shown.

On Fig. 30 three curves are also indicated from Moncrieff's formulae. These are undoubtedly much more accurate than the L.C.C. figures, and it will be seen that they introduce the idea of flat-ended stanchions. The formulae are altogether to be preferred, as giving more reasonable stresses for short stanchions where buckling does not arise, and in that the shape of the curves conforms much more accurately to experimental work.

If a curve of a simpler form than Moncrieff's is desired for practical steelwork design, remembering that stanchions are neither fixed nor rounded as to their ends, the author would much prefer the straight line formula, shown in Fig. 30 by a thick line, and would have no hesitation in using this for any practical steelwork stanchion with the ordinary riveted connexions at the top and bottom. This line has the simple formula

$$\text{safe stress} = 7\tfrac{1}{2} - \frac{l}{30\,r} \text{ tons per sq. in.}$$

It will be seen that it lies everywhere on the safe side of the Moncrieff formula with flat ends, but lies outside the L.C.C. formulae, coinciding approximately with the L.C.C. formula for fixed ends at the lower portion of the diagram, but giving somewhat higher results at the higher portion of the diagram.

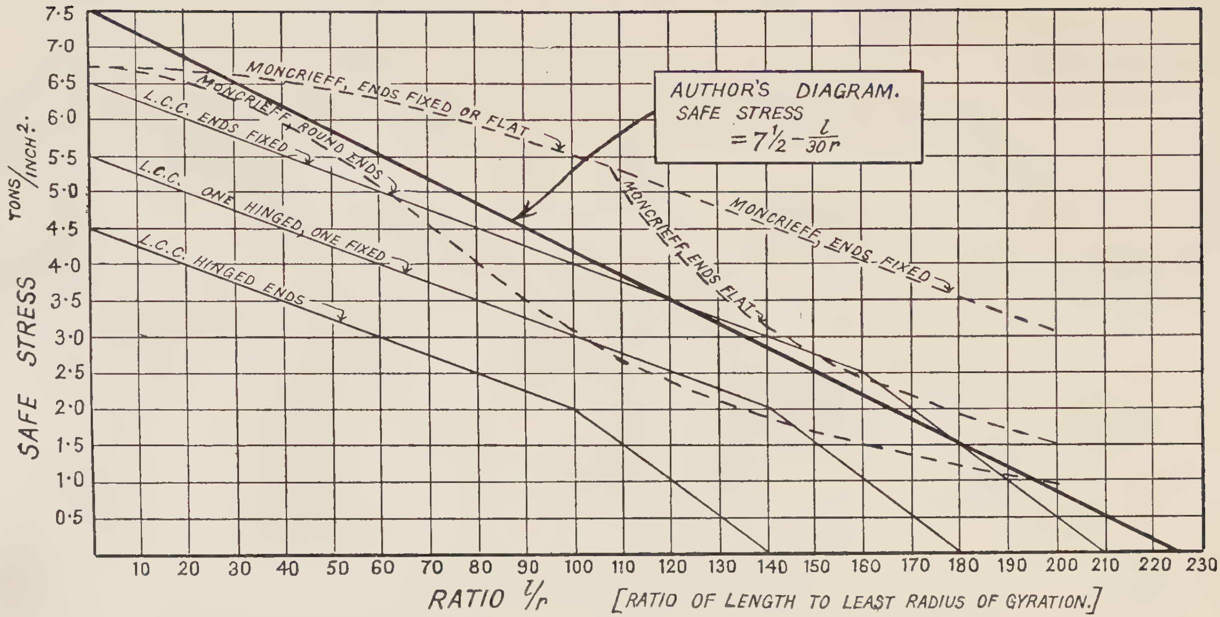


FIG. 30. Safe Stresses in Stanchions.

By Moncrieff, L.C.C., and author's diagram.

The writer thinks there is no doubt that if $7\frac{1}{2}$ tons is safe for the compression flanges in girders it must also be accepted as permissible for very short stanchions where the question of buckling does not arise. This simple formula is reproduced again in Fig. 31 free from the distraction of the other curves for more convenient use.

It will be noticed that the safe stress is stated in these diagrams not in terms of the ratio of length to diameter or thickness, but in terms of the ratio of length to a quantity denoted by r, called the least radius of gyration. The reason for this is the following.

If we make a series of experiments on stanchions to ascertain the safe load for various slenderness ratios for columns of various sectional forms, such as circular, rectangular, joist section, and so on, we find that if we denote the slenderness ratio in terms of length divided by diameter we get different curves for the different sections, but that if we put the curve in terms of length divided by a quantity r, called the radius of gyration, then one curve will serve for all stanchion forms, which is obviously a matter of convenience.

The radius of gyration itself is a somewhat complicated mathematical entity and is expressed by

$$r = \sqrt{\frac{I}{A}}$$

where I is the moment of inertia of the section and A the cross-sectional area. The moment of inertia is a quantity perhaps best excluded from any treatment which pretends to be simply explained, and therefore this side of it will not be followed up in this treatise.

For our present purposes it will suffice to know that the radius of gyration has the following value :

(*a*) *In solid rounds.*

$$r = \cdot 25\ d$$

where d is the diameter.

(*b*) *In solid rectangles.*

$$r = \cdot 289\ d$$
$$\text{or} \qquad \cdot 289\ b$$

where d and b are the depth and breadth respectively, and the lower value is to be taken.

(*c*) *In plain joist stanchions.*

$$r = \cdot 2\ b \text{ approximately}$$
$$= \cdot 4\ d \text{ approximately}$$

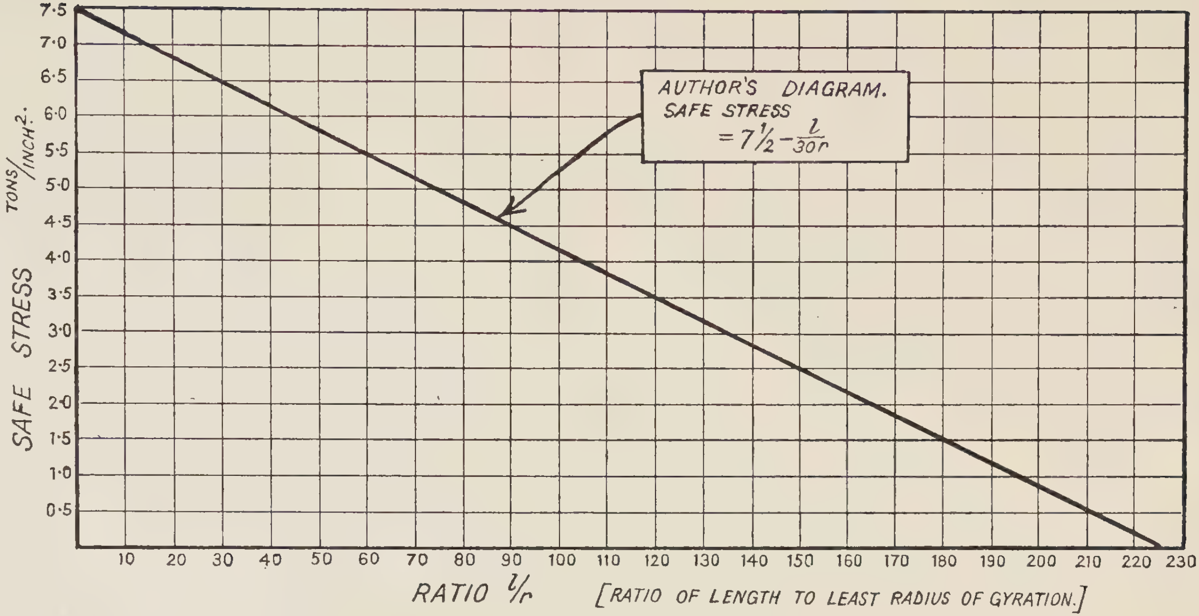


FIG. 31. Safe Stresses in Stanchions.

By author's diagram.

where b and d are the breadth and depth respectively. The lower of these values of r is to be taken.

(*d*) *In plated joist stanchions.*

$$r = \cdot 23\,b \text{ approximately}$$
$$= \cdot 43\,d \text{ approximately.}$$

These values enable us for all practical purposes to state at once the radius of gyration for any practical shape of stanchion.

In the case of solid circles and rectangles the values are accurate and in the other cases are very approximate.

For example, a $24 \times 7\frac{1}{2}$ joist would give

$$\begin{aligned} r &= \cdot 2\,b \\ &= \cdot 2 \times 7\tfrac{1}{2} \text{ in.} \\ &= 1\cdot 5 \text{ in.} \end{aligned}$$

where the exact value is given by the tables as 1·51.

Similarly, the other radius of gyration is given by

$$\begin{aligned} r &= \cdot 4\,d \\ &= \cdot 4\,d \times 24 \text{ in.} \\ &= 9\cdot 6 \text{ in.} \end{aligned}$$

as against 9·5 in. given by the tables.

In plated joist stanchions we may consider a $24 \times 7\frac{1}{2}$ joist with $14 \times 1\frac{1}{2}$ in. plates on each flange, making a total depth of 27 in. and a breadth of 14 in. Here our formula gives

$$\begin{aligned} r &= \cdot 23\,b \\ &= \cdot 23 \times 14 \text{ in.} \\ &= 3\cdot 21 \text{ in.} \end{aligned}$$

as compared with 3·24 in. given by the tables, and

$$\begin{aligned} r &= \cdot 43\,d \\ &= \cdot 43 \times 27 \text{ in.} \\ &= 11\cdot 6 \text{ in.} \end{aligned}$$

as compared with 11·53 in. given by the tables.

The reason stanchions generally have two radii of gyration is because the stanchion is generally more liable to bend in one direction than it is in the other, and the two radii correspond to the two directions. In practice we adopt the lower value, as the stanchion will fail in the weaker direction.

As an example of the use of the table, Fig. 31, let us take the case of the stanchion previously referred to, consisting of a $24 \times 7\frac{1}{2}$ joist with $14 \times 1\frac{1}{2}$ plates on each flange, forming a stanchion 27×14 overall.

The least radius is

$$\cdot 23 \times 14 \text{ in.} = 3\cdot 21 \text{ in.}$$

The other radius is

$$\cdot 43 \times 27 \text{ in.} = 11{\cdot}6 \text{ in.}$$

and does not interest us, as we are bound by the lower value of 3·21.

If the stanchion is 12 ft. high between floor levels where it is stayed, its slenderness ratio will be

$$\frac{l}{r} = \frac{144 \text{ in.}}{3{\cdot}21 \text{ in.}}$$
$$= 45 \text{ approximately,}$$

whence from the diagram, Fig. 31, the safe stress is 6 tons per sq. in. As the sectional area is 71·4, the safe load is therefore

$$P = 71{\cdot}4 \times 6$$
$$= 428{\cdot}4 \text{ tons.}$$

We have dealt with the reduction in stress in long stanchions to provide against buckling. We propose now to consider another complication in stanchion design arising from bending or eccentric loading.

Hitherto we have considered stanchions as being concentrically loaded, that is to say, so loaded that every unit of cross-sectional area shares the stress equally. Obviously in symmetrical sections this requires the load to be applied on the centre of the stanchion. In the case of unsymmetrical sections it requires the load to be applied on the centre of gravity of the cross-sectional area.

Girders are generally connected to stanchions by being supported on a bracket riveted to the side of the stanchion either on the flange, as in Fig. 32 (*a*) or Fig. 33 (*a*) (when the connexion is called a flange connexion), or on a bracket riveted to the web, as in Fig. 33 (*b*) (when the connexion is known as a web connexion).

Referring to Fig. 32 (*a*), it is clear that the reaction of the girder will lie somewhere within the limits of the bracket, and as these brackets are generally about 4 in. projection from the edge of the stanchion it is not unusual to consider the reaction as occurring at a distance of 2 in. from the outer face of the stanchion, as shown on Fig. 33 (*a*). If the stanchion is 12 in. square the eccentricity from the centre line of the stanchion is then obviously

$$e = 6 + 2 = 8 \text{ in.}$$

If, on the other hand, the bracket is riveted to the web, as in

Fig. 33 (*b*), it is not unusual to consider the eccentricity as equal to 2 in. from the centre line.

If the single angle bracket is not strong enough to carry the reaction of the girder the bracket is stiffened by having stiffeners, consisting as a rule of vertical angles, riveted to it, the upper end of these angles being so shaped as to fit the underside of the bracket and so give it additional support. Obviously the number of rivets connecting the bracket to the stanchion must be sufficient to resist the reaction without producing excessive shearing stresses in the rivets.

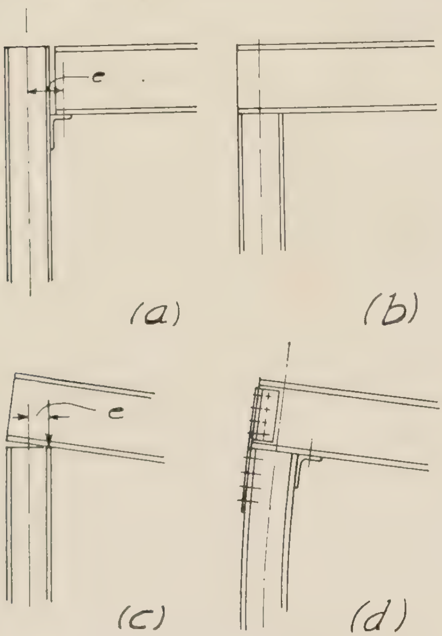


FIG. 32. Illustrating Bending in Stanchions due to Eccentric Loading.

What is at first sight not so clear, but nevertheless just as true, is that even when the girder rests on top of the stanchion, as shown in Fig. 32 (*b*), the load will still be applied eccentrically on the stanchion.

This will be more easily appreciated when we remember that no girder can carry its load without deflecting, and although this deflection may be small owing to the stiffness of the steel of which it is composed, it must be remembered that this applies equally to the stanchion, and that a very small bending in the stanchion corresponds to a considerable bending stress because the stanchion is made of just as stiff a material as the girder.

When the girder deflects one of two things must happen. Either the joint between the beam and the stanchion will open as in Fig. 32 (*c*), when it is obvious that contact will be made near one edge of the stanchion ; or, if the connexion between the girder and the stanchion is made so strong and stiff that this cannot take place, then the stanchion is forced to bend through the same angle as the end of the beam. But to bend a

structural member requires the application of a bending moment to it, and therefore in either case the stanchion is subject to a bending moment in addition to its load.

We therefore see that, much as we might desire to believe that stanchions were generally approximately concentrically loaded, because of the simplification in our calculations which would result, we have to face the fact that, in practice, concentric loading of stanchions is exceedingly rare, and as stresses in stanchions due to comparatively small eccentricities are quite considerable compared with the direct stresses (i. e. the total load divided by the total area), the error involved in assuming the loading to be concentric, when in fact it is not so, is an error of considerable magnitude, and one which results in a considerable reduction in the factor of safety desired.

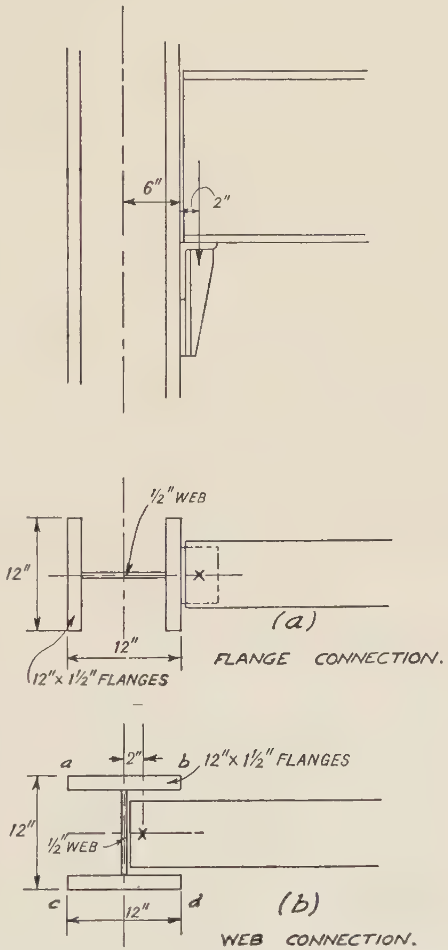


Fig. 33. Details of Stanchion Connexions.

It is therefore quite necessary that this complication should be frankly faced, and if we want to make simplifications for our greater convenience where we neither have time nor perhaps ability to do the work with the utmost accuracy, we must see that our approximation is one which errs on the side of safety.

The stanchions in the interior of a building frequently have

girders symmetrically disposed about them in such a way that the eccentricities of the various girders would appear to be balanced on the stanchion. In this case the eccentricity is generally as low as is met with in any practical case, but even in this case it would not be true to call the stanchion quite concentrically loaded, because in our buildings it is necessary to make provision for all cases, for the floors on one side of a stanchion may be fully loaded while those on the other side may be unloaded, or only partially loaded, and a little consideration will show that under those circumstances the eccentricities of the girders on the two sides will only partially balance each other, and a residual eccentricity or bending moment will remain.

By way of learning to appreciate problems involved let us first take the simple case of a girder simply supported on a flange bracket, as in Fig. 33 (a), without a cleat connecting the top of the girder to the stanchion, and we will assume the top of the stanchion free, so that the whole bending moment has to be resisted at the section below the bracket.

It can be shown both mathematically and experimentally that a load W at an eccentricity e produces the same stresses as the combined effects of the same load W acting on the centre line of the column and a bending moment of

$$M = We$$

applied to the column. We can therefore calculate the stress to these two straining actions separately and superimpose them. They are both cases with which we have already learnt how to deal separately.

The first produces what we call the *direct stress*, which is simply the load divided by the area. The second produces the *bending stress* which is given by the bending moment divided by section modulus, and which is calculated exactly as we have already done for beams.

The direct stress is a uniform stress over the whole section of the stanchion, while the bending stress, as we have already seen for beams, is one which varies from a maximum compressive stress on one edge to a maximum tensile stress at the other edge, the stress on the neutral axis (which coincides with the centre line in symmetrical sections) being zero, as was the case for beams.

Let us illustrate this by a simple example. The solution given is an approximate one only, sufficiently accurate for

most practical purposes, and on the same lines as that already given for beams.

We will select the case illustrated in Fig. 33 (*a*), in which the stanchion is 12 in. square, the flanges being 12 in. $\times$ $1\frac{1}{2}$ in. plates, and the web 9 in. $\times$ $\frac{1}{2}$ in. thick. For the sake of simplicity the complication of the interconnecting angles or joist flange is omitted, though no difficulty would be found with this, as it follows exactly the treatment already given in the case of girders. Our approximate solution would be as follows:

Sectional area.	sq. in.
Two flanges 12 in. $\times$ $1\frac{1}{2}$ in.	$= 36$
One web 9 in. $\times$ $\frac{1}{2}$ in.	$= 4\frac{1}{2}$
Total area	$= 40\frac{1}{2}$

We will assume the load on the girder is 40 tons, the reaction on this bracket being 20 tons. Therefore the direct stress is

$$f_1 = \frac{W}{A} = \frac{20 \text{ tons}}{40\frac{1}{2} \text{ tons}} = \cdot 5 \text{ ton per sq. in. approximately.}$$

The section modulus, as we have already seen, is given approximately by multiplying the area of one flange by the distance from the centre of one flange to the centre of the other, so that

$$Z = A \times d = 18 \text{ sq. in.} \times 10\tfrac{1}{2} \text{ in.} = 189 \text{ in. units.}$$

The bending moment is

$$M = We = 20 \text{ tons} \times 8 \text{ in.} = 160 \text{ in. tons.}$$

Therefore the stress due to bending is

$$f_2 = \frac{M}{Z} = \frac{160}{189} = \cdot 85 \text{ ton per sq. in. approximately.}$$

The maximum stress is therefore

$$f_1 + f_2 = \cdot 5 + \cdot 85 = 1\cdot 35 \text{ tons per sq. in.}$$

It will be seen that the stress, taking eccentricity into account, is in this case nearly three times the stress if this eccentricity were ignored, and it will, of course, be obvious that this is not a matter which can properly be ignored.

Let us now make the somewhat similar calculation for the web connexion shown in Fig. 33 (*b*). Clearly the load and the cross-sectional area remain unaltered, and therefore the direct stress is

$$f_1 = \cdot 5 \text{ tons per sq. in.,}$$

as before. The section modulus in this case is, however, quite different, and much less than the value we took before, because the eccentric loading illustrated produces bending of the stanchion section in its weak direction. The stanchion is, in fact, bent in the same way as a joist would be if we placed it between two supports with its web horizontal instead of vertical as usual. Clearly under these circumstances the tips of two of the flanges would be in compression and the other tips of the flanges would be in tension, and it is obvious that the section would be very much weaker in this direction.

Referring to Fig. 33 *b*, the corners *b* and *d* would be in compression and the corners *a* and *c* would be in tension, the web lying practically on the neutral axis being unstressed by the bending action.

It is clear that if the web is not subject to stress it can be omitted in the calculation of strength, and in fact a section of this kind bent in its weak direction is exactly equivalent in strength to the two flanges treated separately as rectangular members, having a depth of 12 in. and a breadth of $1\frac{1}{2}$ in. each, the whole section, therefore, being equivalent to one rectangular section 12 in. deep and 3 in. broad, for which we have already seen that the section modulus is

$$Z = \frac{bd^2}{6} = \frac{3 \times 144}{6} = 72 \text{ in. units.}$$

The bending moment in this case is

$$M = We = 20 \text{ tons} \times 2 \text{ in.} = 40 \text{ in. tons,}$$

whence the bending stress is

$$f_2 = \frac{M}{Z} = \frac{40}{72} = \cdot 55 \text{ ton per sq. in. approximately,}$$

whence the maximum stress is

$$f_1 + f_2 = \cdot 5 + \cdot 55 = 1 \cdot 05 \text{ tons per sq. in. approximately.}$$

From this the importance of considering the effect of eccentricity is brought out as before.

It is also of interest to notice that, although the bending moment with the flange connexion was four times as great as that with the web connexion, yet the resultant stresses are not very different, owing to the large bending moment in the flange connexion being resisted by the large section modulus, while the small moment with the web connexion occurs with a correspondingly small section modulus.

The distribution of stress across the flange connexion is

illustrated in Fig. 34. We set up above the base of our stress diagram a constant stress of ·5 ton per sq. in., representing the direct stress. In addition to this we have the bending stress of ·85 ton per sq. in. at the two flanges, being, however, of opposite sign as drawn, and passing through the neutral axis

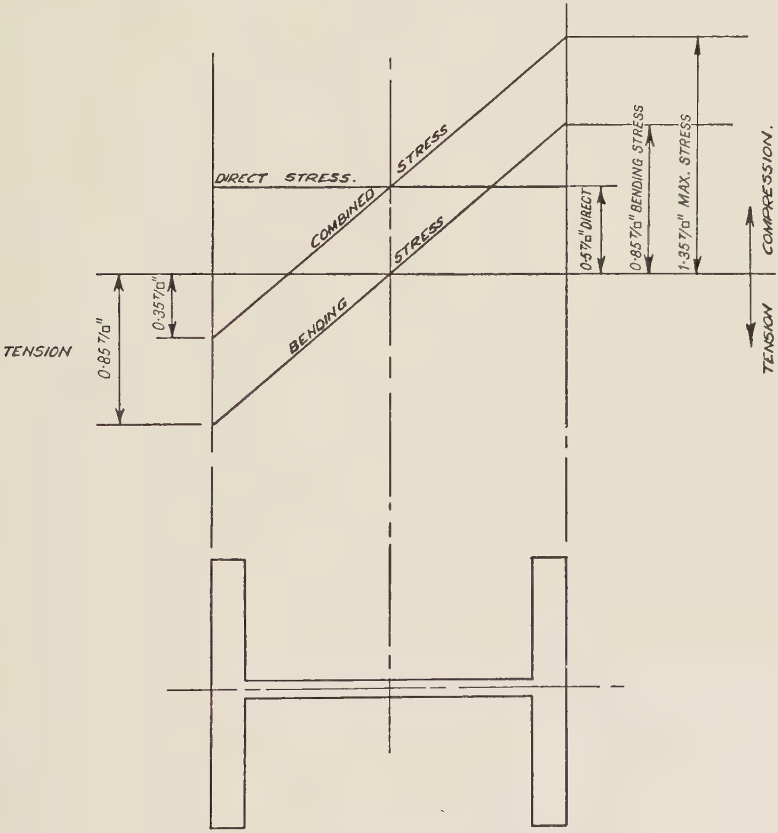


Fig. 34.

at the point of zero stress. Adding these two diagrams together, we get the curve of resultant or maximum stress, giving 1·35 tons per sq. in. as the maximum stress on one side in compression, and a maximum stress of

$$\cdot 85 - \cdot 5 = \cdot 35 \text{ tons per sq. in.}$$

in tension on the other side of the stanchion.

A similar diagram could be drawn for the web connexion, but it is thought that the matter is sufficiently clear.

We have now dealt with the treatment of eccentric loading on a stanchion owing to the seating being on a flange bracket or a web bracket, the combined stress in either case being obtained by adding the stress due to the direct load to the stress due to bending. In practice our examples are generally considerably more complicated, as a stanchion generally carries more than one beam at each floor level, and it frequently happens that a flange connexion not only has the eccentricity due to the bracket, but the bracket may be moved sideways along the flange so as no longer to be on the centre line of the web. Reverting to the same stanchion as we considered in Fig. 33 the stress obtained with one flange connexion with the 20 tons reaction and one web connexion of the same amount would be as follows:

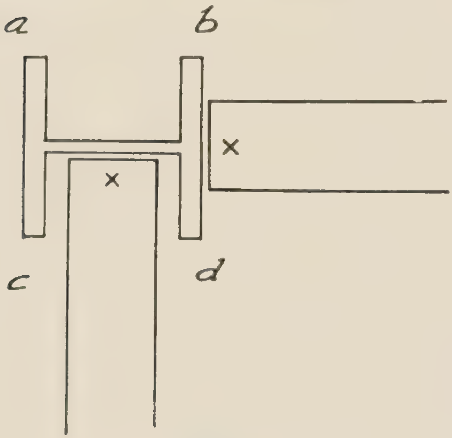


FIG. 35.

Direct stress.

$$f_1 = \frac{40 \text{ tons}}{40\tfrac{1}{2} \text{ sq. in.}} = 1{\cdot}0 \text{ ton per sq. in. approximately.}$$

Bending stress due to flange connexion.

$$f_2 = \frac{160}{189} = {\cdot}85 \text{ ton per sq. in.}$$

Bending stress due to web connexion.

$$f_3 = \frac{40}{72} = {\cdot}55 \text{ ton per sq. in.}$$

$$\text{Total} \quad 2{\cdot}4 \text{ tons per sq. in.}$$

This maximum stress would occur at the corner d in Fig. 35. At the corner a there would be a tension amounting to

$${\cdot}85 + {\cdot}55 = 1{\cdot}4 \text{ tons per sq. in.,}$$

less the direct compression stress of 1 ton per sq. in., leaving a net tension stress of 0·4 ton per sq. in.

It frequently happens that four beams are carried at each floor level, though the reactions may be of different amount. Thus, Fig. 36 illustrates such a case in which the reactions are figured on. Referring first to the flange connexions it will be seen that the 10 tons on the left-hand side partially balances the 20 tons on the right, leaving only a bending moment of the remaining 10 tons multiplied by its proper eccentricity. Similarly the 30 tons at the top of the diagram is partially balanced by the 15 tons at the bottom, leaving only a bending moment of 15 tons multiplied by its proper eccentricity to be resisted. The stress due to these two bending moments would therefore be calculated and added to the stress due to the total load, which in this case amounts to 75 tons. The bending moment due to these eccentricities is generally considered to have died out at the floor below, so that at this floor the load on the column from the upper floors is generally taken as concentric, and the only bending moments are those due to the eccentricities of the beams at that particular floor level, so that while the column loads gradually increase as we proceed downwards from floor to floor, the bending moments in the columns are frequently substantially the same at all floor levels.

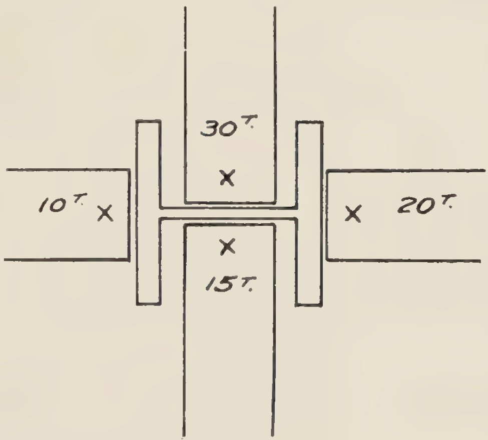


Fig. 36.

There are a good many matters in connexion with this subject, of which a full treatment would be out of place in this book; but it may not be out of place to mention the points so that readers at any rate may know of their existence, even if they hardly feel competent to supply a correct mathematical solution. The first refers to the eccentricity of beams on columns. In the case of a beam sitting on a bracket, as shown in Fig. 33, the treatment given is substantially accurate, though the exact point of application of the load may vary to some extent from the centre of the bracket: but this is a small

discrepancy. It will, however, be noticed that in this diagram the top flange of the girder is shown unconnected to the stanchion, so that the stanchion and the beam are free to deflect or bend according to their separate bending moments without the one member putting any restraint upon the other, and the treatment given is only approximately true for this case.

In practice it is usual, for several reasons, to connect the top flange of the girder to the stanchion by means of a top cleat, as shown in Fig. 37. This construction makes the whole framework much more rigid and able to resist wind pressure and other lateral forces; whereas obviously the construction shown in Fig. 33 would be comparable to a hinged connexion and the structure would be free to sway unless otherwise prevented.

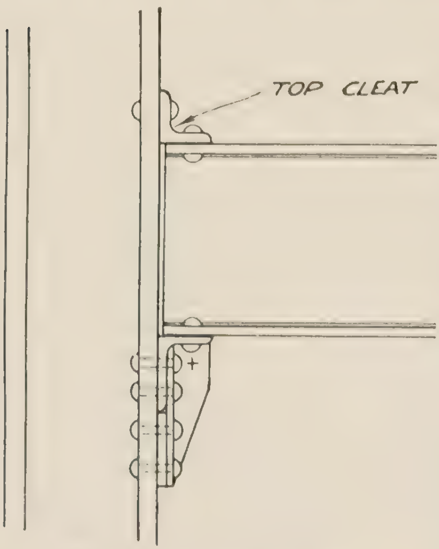


Fig. 37. Showing Top Cleat.

The top cleat in the connexion, though on many grounds desirable, undoubtedly very much complicates the question as to what is the proper bending moment or eccentricity on the column. If the beam, under the action of its load, is tending to deflect more than the column, as shown in Fig. 38, the effect will be that the rivets or bolts connecting the top cleat to the stanchion will be put into a state of tension represented by a force P, and this force P, multiplied by the distance d, is an additional bending moment producing eccentricity over and above that already given in the previous treatment. The possible value of this additional moment was frequently in practice limited by the strength of the top cleat or the rivets connecting it. Thus, if the cleat is connected by two $\frac{3}{4}$ in. rivets having a safe load of 2·4 tons each, the safe load on the rivets would be 4·8 tons. The yield point of the rivets is generally between 2 to $2\frac{1}{2}$ times the safe load, so that $2\frac{1}{2}$ times 4·8 tons would at any rate appear to be the limiting value of P,

and the additional bending moment, calculated on this basis, would at any rate be on the side of safety. It does not follow however, that the introduction of the top cleat will always increase the bending moment of the stanchion. It sometimes happens that the stanchion is more flexible, and likely to bend at the joint more than the beam, in which case making a rigid connexion between the stanchion and the beam may have the effect of introducing a compression at the point P, as shown in Fig. 39. In this case the bending moment due to the stiffness of the joint ($M = Pd$) is clearly of opposite sign to the bending moment due to the eccentricity of the beam on the stanchion ($M = W \times e$).

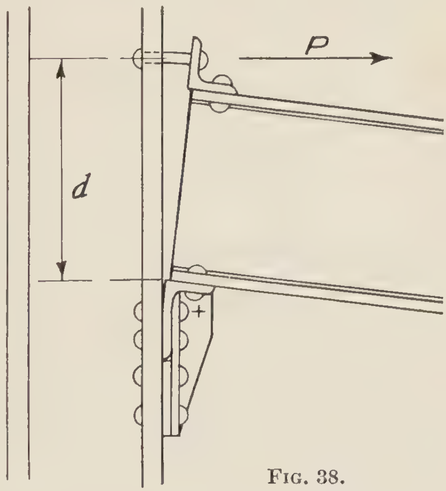


FIG. 38.

The mathematical treatment of this matter is unfortunately not a very simple one, and the reader wishing to proceed further with it is recommended to study the treatment of bending moments in reinforced concrete columns monolithic with beams, given in a companion volume, *Reinforced Concrete Simply Explained*,[1] the problem being exactly analogous. He will find that the eccentricity is great in cases where the column is stiff in comparison with the beam and small in the contrary event, the stiffness of a member being proportional to its moment of inertia divided by its length.

The next matter requiring to be at least mentioned is that the treatment of eccentricities or bending moments in columns, illustrated in Fig. 33, assumed that the whole bending moment had to be resisted by one column section. In actual practice this rarely obtains except at the extreme top of a stanchion, where it carries the beams from the roof and does not continue higher. At most of the lower floors the stanchion is continuous above and below the floor section, and the bending moment introduced by the beams is generally shared in the stanchion by

[1] Oxford University Press, 2nd ed., 5*s*. net.

the section immediately above and immediately below the floor. Where the story heights are equal and the stanchion goes up with constant section this moment will be shared equally by the section immediately above and below, so that the bending moment in such cases would be one-half the total moment introduced by the beams from that floor; but when the story heights are unequal and the stanchion changes its

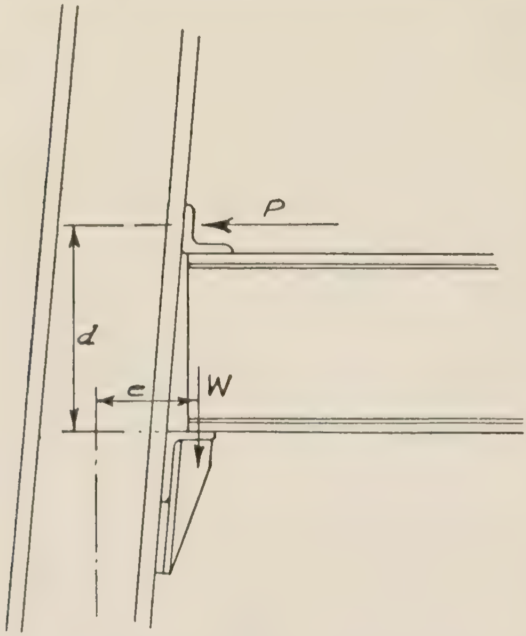


Fig. 39.

section at a floor, the upper and lower sections of the stanchion will share the total bending moment in proportion to their stiffness, as defined previously. It is unnecessary to pursue the matter further in the present treatise.

Fig. 40 shows the bending-moment diagram in a stanchion eccentrically loaded by beams in the various floors, from which it will be seen that the bending moment is greatest at the floor and dies away to zero at some point, generally about half-way between the floors.

It sometimes happens that a stanchion receives a bending moment or eccentric load at some point intermediate between its supports, as, for example, in the case of a stanchion carry-

ing a bracket to receive a travelling crane, as shown in Fig. 41. In such cases if We is the bending moment exerted by the bracket this will be shared by the upper and lower sections of columns in proportion to their length. This is easily done graphically by setting out the length $a\ b$ and $c\ d$, equal to the whole moment We, joining $a\ d$ and $b\ c$, and drawing a horizontal line into the diagram at the level of the bracket intersecting

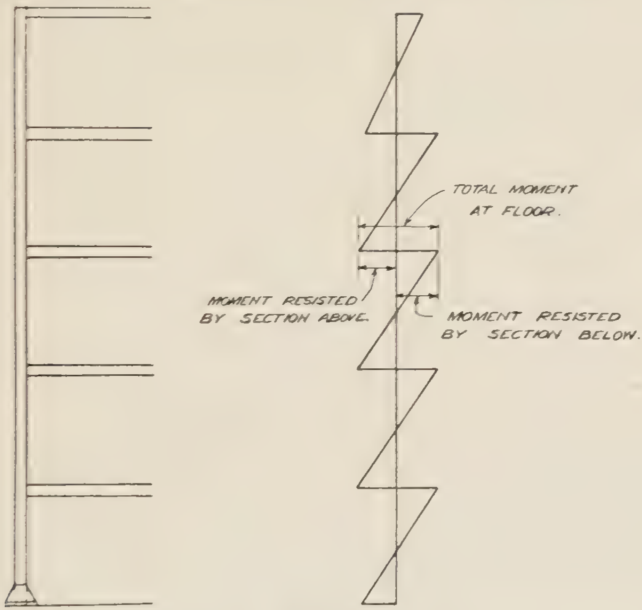


Fig. 40.

$a\ d$ in p and $b\ c$ in q, the broken line $a\ p\ q\ c$ then being the bending-moment diagram, the maximum value being $p\ r$ or $r\ q$, whichever is the greater. This treatment is only correct when the ends of the columns are free, but the solution in other cases is too complicated for simple treatment.

Stanchions are also subject to bending moments due to racking owing to the effect of wind and other horizontal forces; indeed, in some structures, notably in travelling cranes and water towers, these bending moments produce the greatest stresses which the stanchion is called upon to resist.

The student is therefore warned that any simple treatment including a consideration of these matters should be looked at with some caution, as it may well be simple only in appearance

and at the expense of an approximation to the actual conditions arising. Unfortunately, the calculation of the straining actions, which has been only lightly touched upon in this chapter, is not calculated on the side of safety, and those who ignore these matters ought at any rate to be quite sure that their factor of safety is large enough to make adequate provision for them.

A prudent designer makes a point, when there is a doubt, of erring on the side of safety, and only those who can eliminate

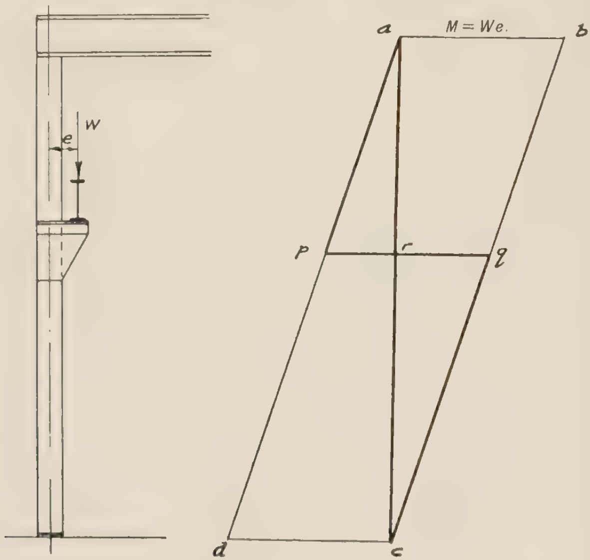


FIG. 41.

the doubt by assigning accurate values to the stresses due to all these various straining actions can safely produce the most economical design. In these matters we must secure safety at all costs. If on this we can superimpose the ability to produce a really accurate analysis of stresses, then our safety can be obtained with economy.

When the stresses from all these straining actions have been properly computed, the maximum stress is to be compared with the safe stress for the particular slenderness ratio, taken from an appropriate diagram, such as Fig. 30 or Fig. 31. The design of stanchions (like much other engineering design) therefore becomes a matter of trial and (where necessary) of correction, though an experienced engineer will make a very near trial first time.

CHAPTER VIII

RIVETED AND BOLTED CONNEXIONS

STRUCTURAL members are connected by fasteners containing either rivets or bolts. The ordinary rivet has a snap head, as shown in Fig. 42. A hole is drilled through the section requiring to be riveted, and the rivet, as in Fig. 42 (*a*), is inserted, having previously been raised to a red heat. It is then ‘ closed ’ until it adopts the form in Fig. 42 (*b*). In works this ‘ closing ’ is generally done by a hydraulic press, taking the form of a heavy casting of cranked form supporting two

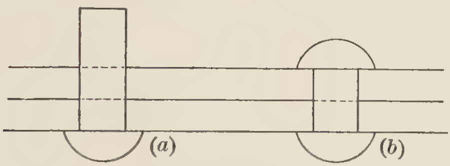


FIG. 42. Snap-Headed Rivets.

cup-shaped jaws, one fixed and the other so arranged that it can be pressed towards the other with a force of about 20 tons exerted by hydraulic pressure, the gap between the two being sufficiently wide to enable ordinary structural sections to be inserted.

On the site this method of closing rivets has practical difficulties, as it requires hydraulic pumps, accumulators, and piping to the various portions of the job, and the hydraulic riveter itself is naturally a heavy casting, which can only be carried by a crane. For this reason rivets on site are generally closed either by hand or with a pneumatic hammer. In either case a large number of blows are delivered on to a cup-shaped tool on the shank of the rivet, the head being, of course, held by a heavy hammer on the other side.

The rivet expands considerably when heated, and it is usual to make the holes $\frac{1}{16}$ in. larger in diameter than the rivet, so that it will easily pass through when hot : thus a $\frac{3}{4}$-in. rivet has $\frac{13}{16}$-in. holes. When the rivet is closed the first action is to shorten the rivet and make it expand laterally so that it completely fills the hole. After that the energy goes in forming the

head. The rivet then cools and, in cooling, contracts and becomes shorter. In so doing it naturally grips very tightly the plates which it is riveting together and produces a great deal of friction between them, which resists relative movement of the plates, quite apart from the shearing strength of the rivet itself.

If we rivet two plates together, as in Fig. 43, and subject these plates to tension in a testing machine, failure occurs by shearing of the rivet, the rivet after failure taking up the form as in Fig. 43 (*b*). It is found that resistance to failure of this kind is directly proportional to the cross-sectional area of the rivet.

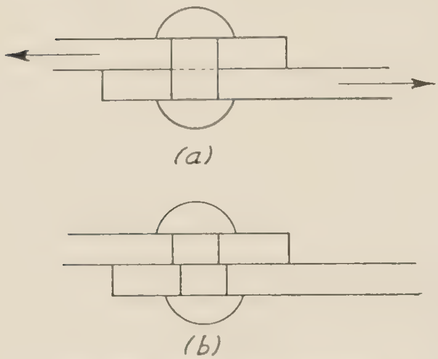


FIG. 43. Rivet in Single Shear.

For rivet steel we use a relatively soft material, in which the safe shear strength may be taken at $5\frac{1}{2}$ tons per sq. in., so that a $\frac{3}{4}$-in. rivet, having a sectional area of ·44 sq. in., will have a shear strength of

$$\cdot 44 \times 5\tfrac{1}{2} = 2{\cdot}4 \text{ tons.}$$

This takes no account of the friction between the plates which is set up by the tension in the rivet produced by its shortening. The amount of this tension depends on a variety of factors, including the temperature of the rivet when it was closed, whether the plates were touching hard or separated by a small space, as sometimes happens owing to a certain spring in them, and whether the rivet was kept closed in the hydraulic ram for a second or two during cooling, which tends to increase the tension in it. If we take the tension in the rivet at 11 tons per sq. in. and the coefficient of friction as $\frac{1}{4}$, then for each square inch of rivet there will be an additional 2·75 tons resisting movement of the plates owing to the friction between them. In this country it is not usual to make any allowance for this friction, but it often accounts for considerable discrepancies between the strength of riveted joints when tested as compared with the strength that we should expect them to give on a basis of the shear strength of the rivet alone.

In the case of rivets which are very long compared with their diameter the shrinkage of the rivet when cooling may produce

tensile stresses so great as to break the rivet, and it is unusual to use a rivet longer than five times its diameter. For greater lengths than this bolts carefully fitted to the holes are generally substituted in first-class work.

The rivet in Fig. 43 will break on a single section, and is therefore said to be in *single shear*. Occasionally, as in Fig. 44, a rivet may be in double shear if the two outer plates are pulling in one direction and the middle plate pulling in the opposite direction. Such a rivet after failure would be as in Fig. 44 (*b*). It requires approximately twice the effort to break a rivet in double shear as compared with single shear, but most regulations require that the strength of a rivet in double shear shall be confined to $1\frac{3}{4}$ the strength in single shear, so that a $\frac{3}{4}$-in. rivet in double shear would carry safely

$$2{\cdot}4 \times 1\tfrac{3}{4} = 4{\cdot}2 \text{ tons.}$$

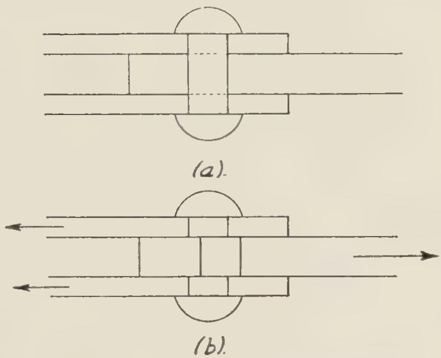


FIG. 44. Rivet in Double Shear.

If relatively large rivets are used to rivet together relatively thin plates it may happen that the force required to shear the rivet is greater than the pressure which the plate will take against the area of the hole without failure of the plate ; thus, in Fig. 44 (*b*) if the centre plate is $\frac{1}{2}$ in. thick and the rivet $\frac{3}{4}$ in. the rivet would bear against an area of

$$\tfrac{3}{4} \text{ in.} \times \tfrac{1}{2} \text{ in.} = \tfrac{3}{8} \text{ sq. in.}$$

The total force of 4·2 tons on the rivet would therefore have to be carried on this area, producing a bearing stress of

$$\frac{4{\cdot}2 \text{ tons}}{\frac{3}{8} \text{ sq. in.}} = 11{\cdot}2 \text{ tons per sq. in.}$$

Most regulations require this stress to be kept within the limit of 11 tons per sq. in. This is, of course, greater than we normally use on steel in compression, but it is recognized that, where an intense stress is very local in its action and disperses rapidly, a greater stress can be allowed safely. Cases where bearing pressure becomes the limiting factor are more likely to occur with rivets in double shear than when they are in

single shear, because the load on the rivet, and therefore on the plate, is proportionately greater.

The holes in plates to receive the rivets used to be made by punching. A plate, as in Fig. 45, would be supported on a die containing a circular hole, above which a punch somewhat smaller in diameter than the hole would be applied, and would be forced through the plate, taking with it a piece of the material from the plate, and leaving a hole through the plate of a

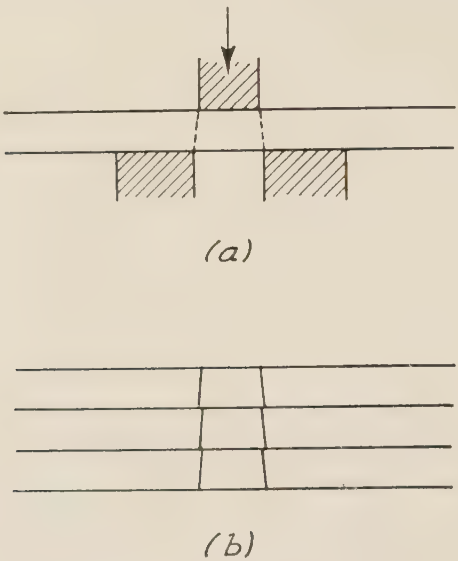


FIG. 45. Punched Holes.

slightly conical form. In work of low quality the author has seen several punch-holes assembled, as shown in Fig. 45 (*b*), with the object of riveting without any further preparation. Even if it were possible to make a rivet fill such a hole a rivet of this shape would be much weaker, because it is found that a member in tension is much weakened by sudden variations of sectional area, the curves of stress requiring to flow in gentle stream lines if high loads are to be carried, and any sudden deflection of stress being always accompanied by an intense concentration of stress.

It is also found that the process of punching, which of course produces intense overstraining at the surface of the punched hole, causes intense hardening of the material round this hole,

which therefore loses the valuable properties of ductility possessed by the remainder of the section, so that when the specimen is tested in a testing machine the rest of the section begins to yield, but the hard steel surrounding the hole is unable to yield, and therefore tends to carry the whole load until it breaks down with the formation of a crack.

It is found, in fact, that the hardening of the steel round the hole due to the process of punching makes a steel section much weaker, and for this reason, wherever punching holes have been used, it is required in good practice that the material round the punched hole shall be removed by the process of reamering. Old-fashioned practice in assembling the various plates that go to make up one flange of a girder was to prepare

Fig. 46. Non-Alignment of Holes due to Separate Drilling of Plates.

a wooden template with the holes drilled, and from this to make the various plates individually, the holes in these plates then being individually made either by punching or drilling, and the plates subsequently assembled on top of each other. It was therefore quite common to find that while there was a general alignment of the holes, considerable variations of an irregular nature might occur, as shown in Fig. 46. This would, in extreme cases, be improved by reamering, but the result can never be a really satisfactory one.

In high-class modern work structural members of this kind are drilled through all the plates in a single operation. The usual tool for this purpose is a set or battery of radial drilling machines, enabling a single man to watch and control the drilling of about six holes through the same girder simultaneously. With this process all the plates in the girder are assembled together and clamped, and the drills then pass through all the plates in a single operation. Besides making a very much better job it is found to be no more expensive in the long run than the other process.

When the structural members are first erected a temporary connexion is made with the field bolts, which in good work are

subsequently removed a few at a time and replaced with rivets. For some classes of work bolts are used permanently for the field connexions in place of rivets, and it is frequently more convenient to the steel contractor. Bolts are, however, not so strong as rivets of the same size, nor do they give to a job the same strength and rigidity as is obtained with rivets.

As compared with rivets bolts have the following disadvantages :

(1) The ordinary black commercial bolt does not fit the hole.

(2) Whereas a rivet is of full section from end to end, a bolt is reduced very considerably in size at the bottom of the thread. Thus, a $\frac{3}{4}$-in. bolt, having an area of ·44 sq. in., only has an area of ·3 at the bottom of the thread. The disadvantage of bolts as compared with rivets is increased in cases where a load has to be taken by a group of bolts or rivets.

In the case of rivets which accurately fill their holes the load will generally be shared between all the rivets of a group equally, whereas with bolts which do not fill their holes it will generally be found that some of the bolts make contact and begin to carry the load before others, so producing unequal distribution, and therefore an increased stress on the bolts which carry the load first, which may result in progressive failure. For these reasons bolts are much inferior to rivets for structural work, and the author would consider a 1-in. bolt required to replace a $\frac{3}{4}$-in. rivet. It must also be remembered that a $\frac{3}{4}$-in. rivet really becomes $\frac{13}{16}$ in. diameter, though no allowance is usually made for this in the calculation of its strength. Obviously no such margin is available in the case of bolts.

While it is desirable that the size of rivets should have regard to the size of the members connected, it is, for practical reasons, of great importance to standardize a job for a single-size rivet as far as possible, as the riveters proceeding from joint to joint would be seriously inconvenienced if they had to carry appliances for many sizes of rivets and pick out rivets of various sizes from the same stove for feeding various joints.

The $\frac{3}{4}$-in. rivet is the one most generally suited to structural work of ordinary size, and is generally adopted as a standard. It is varied from only in the case of such members where there is some strong reason for varying. In some heavy girders, for example, it may be found that the maximum number of $\frac{3}{4}$-in. rivets which can be inserted at the proper spacing, consistent with not too greatly weakening the plates, will not give the

shear strength necessary. In such cases 1-in. rivets are generally adopted.

It frequently happens with large gusset plates in the bases of stanchions where very heavy loads have to be transmitted that the gussets would have to be inordinately long, with $\frac{3}{4}$-in. rivets, and can be reduced in size when 1-in. rivets are adopted. In both these cases the variation from standard takes place with rivets closed at the works, where the objections to varying the size are not quite so great as on the job.

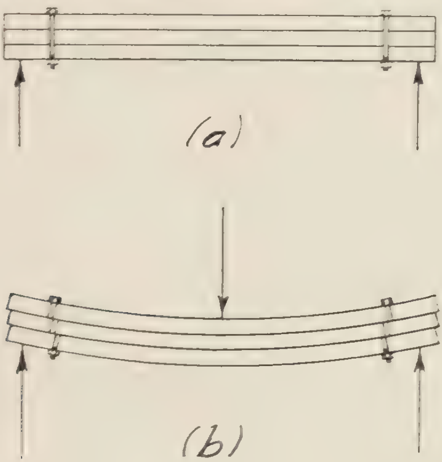


FIG. 47.

We have dealt with rivets and bolts in general, but there are a few special applications which require mention. The first is perhaps the rivets required to connect the flanges to the web of a plate girder.

It was mentioned and proved earlier in this book that vertical shear in a girder is always accompanied by an equal horizontal shear. The existence of this horizontal shear is perhaps made clearer by considering three planks superimposed as shown in Fig. 47 (*a*). Treating these as a beam supported at the ends and loaded in the centre, they would deflect in the manner shown in Fig. 47 (*b*), each plank deflecting equally and separately as shown. It has already been shown that, as each plank forms part of a segment of a circle, the top of each plank, being of smaller radius, will be shorter than the bottom. For the same reason, the bottom of the top plank will be longer than the top of the second plank, and will therefore project

beyond it at each end, as indicated. This means that a bolt or rivet going through each plank would now tend to be broken by the horizontal movement of the planks, as shown in Fig. 47 (*b*), and would tend to take up the shape therein indicated. This illustrates not only the horizontal shear in a girder, but also in the rivets connecting the flanges to the web, because the middle plank may be considered to be the web, the top and bottom planks the top and bottom flanges respectively, the problem being in no way altered in principle by the fact of the web being somewhat thicker than is usual and somewhat less in depth in relation to the thickness of the flanges.

Consider now Fig. 48, showing half of a plate girder in which the flanges are connected to the webs by means of rivets and angles. It is desired to calculate how many rivets are necessary between the points A and B anywhere along the girder, these points being conveniently taken 12 in. apart. There are two ways in which we may proceed.

The first makes use of the fact that the vertical shear and the horizontal shear are equal. If, therefore, we calculate the vertical shear on the plane CC (half-way between AA and BB), and we divide this shear by d, d being measured in feet, then the vertical shear per foot run is $\frac{S}{d}$. This is therefore also equal to the horizontal shear per foot run, and provided sufficient rivets are placed per foot to take up the shear S, our problem will have been solved.

A second method which, of course, yields the same results but is to some people a little easier to follow and is, in any case, a useful confirmation, is to calculate the bending moment at section AA and at BB, the latter being, of course, in the present case the greater. The girder being of constant depth it is equally easy to calculate the flange forces due to these bending moments, P_1 and P_2 respectively, by dividing the bending moments by the depth in each case. Clearly the difference between P_2 and P_1 represents an increment of force in the flange which can only have been transmitted to it through the medium of the shear in the rivets between sections A and B, and therefore sufficient rivets have to be provided between these two sections to take the difference in flange force $P_2 - P_1$.

Let us make this clear by means of an example. Let us take a girder of 30-ft. span with $d = 3$ ft. (measured to the centre of gravity of the flanges), the central load being 60 tons. Let the section AA be 2 ft. from one support and BB 3 ft. from it.

The vertical shear at the section *CC* is clearly 30 tons, and therefore the vertical shear per foot run is

$$s = \frac{S}{d} = \frac{30 \text{ tons}}{3 \text{ ft.}} = 10 \text{ tons per foot run.}$$

This is therefore also the horizontal shear per foot run, and hence sufficient rivets have to be inserted per foot to take this shear.

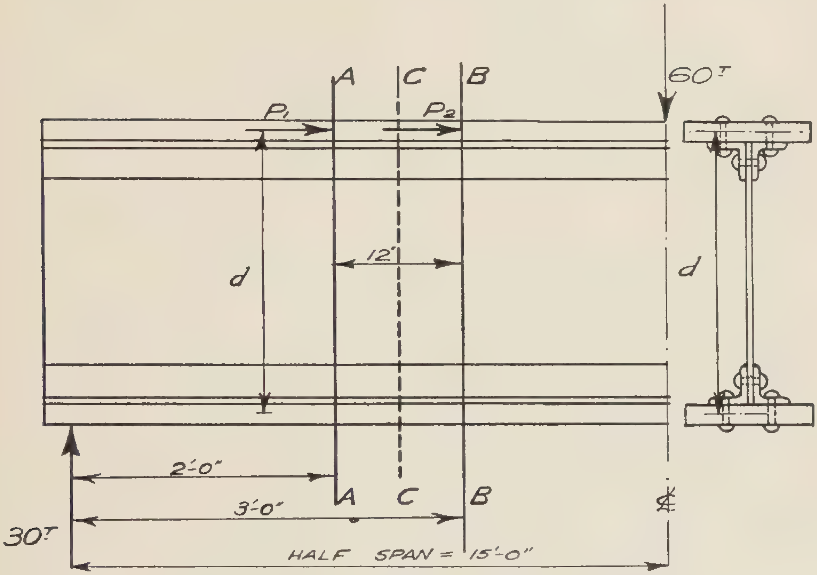


FIG. 48.

Considering first the rivets between the flanges and the angles, there are two of these per row, and each rivet is worth 2·4 tons in single shear if they are $\frac{3}{4}$ in. in diameter, or 4·8 tons per row of two. The rows would therefore need to be approximately 5 in. apart, because this would give a shear per foot of

$$4{\cdot}8 \text{ tons} \times \frac{12 \text{ in.}}{5 \text{ in.}} = 11{\cdot}5 \text{ tons per foot,}$$

as against the 10 tons required.

Coming now to the rivets connecting the angles to the web there is only one of these for each two of the others (assuming they are spaced equally far apart); but they are in double shear, so that the value of each rivet would be

$$2{\cdot}4 \times 1\tfrac{3}{4} = 4{\cdot}2 \text{ tons.}$$

If spaced at 5-in. centres as before, they would therefore be good for

$$4{\cdot}2 \text{ tons} \times \frac{12 \text{ in.}}{5 \text{ in.}} = 10{\cdot}1 \text{ tons},$$

as against the 10 tons per foot required.

Let us now consider the problem, using the second method. The bending moment at AA would be

$$M_A = 30 \text{ tons} \times 2 \text{ ft.} = 60 \text{ ft. tons.}$$

Similarly, the bending moment at BB would be

$$M_B = 30 \text{ tons} \times 3 \text{ ft.} = 90 \text{ ft. tons.}$$

The depth of the girder being 3 ft., the flange forces would be

$$P_1 = \frac{M_A}{d} = \frac{60 \text{ ft. tons}}{3 \text{ ft.}} = 20 \text{ tons},$$

and

$$P_2 = \frac{M_B}{d} = \frac{90 \text{ ft. tons}}{3 \text{ ft.}} = 30 \text{ tons};$$

therefore $P_2 - P_1 = 10$ tons, which, of course, is the horizontal shear per foot, and gives the same value as before.

From the consideration that the horizontal shear per foot is the same as the vertical shear per foot, it follows, of course, that the distribution of horizontal shear along the length of a girder will be exactly the same as that for the vertical shear.

It therefore follows that a girder centrally loaded where the vertical shear is constant from its midspan to the support would also have the horizontal shear constant, and the rivets will therefore require to be uniformly spaced. On the other hand, a girder uniformly loaded has the vertical shear, and therefore also the horizontal shear, varying from zero at midspan to a maximum at the support, and the rivets would therefore require to be closely spaced near the supports and can be further apart as we approach the centre. It is, however, not good practice to put the rivets much more than 6 in. apart, whether the horizontal shear requires them or not.

The number of rivets which can be got from practical considerations generally form the limiting consideration in determining the minimum depth of very heavy short girders, and it will be clear that it would be difficult to exceed 30 tons per foot with a single web.

This gives one some means of determining the minimum depth of girders from a shear consideration. Thus, a girder to

carry 100 tons will have a shear of 50 tons, and should not be less in depth than

$$\frac{50}{30} = 1\tfrac{2}{3} \text{ ft.},$$

and so on, in proportion.

Another problem in connexion with rivets is in the connexions between beams and beams. In Fig. 49 is shown a typical connexion of a secondary beam with a main beam, the connex-

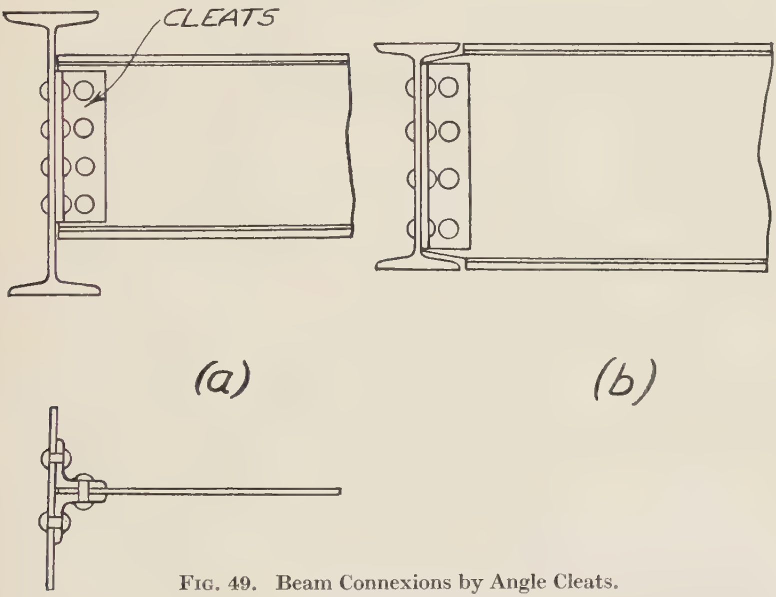


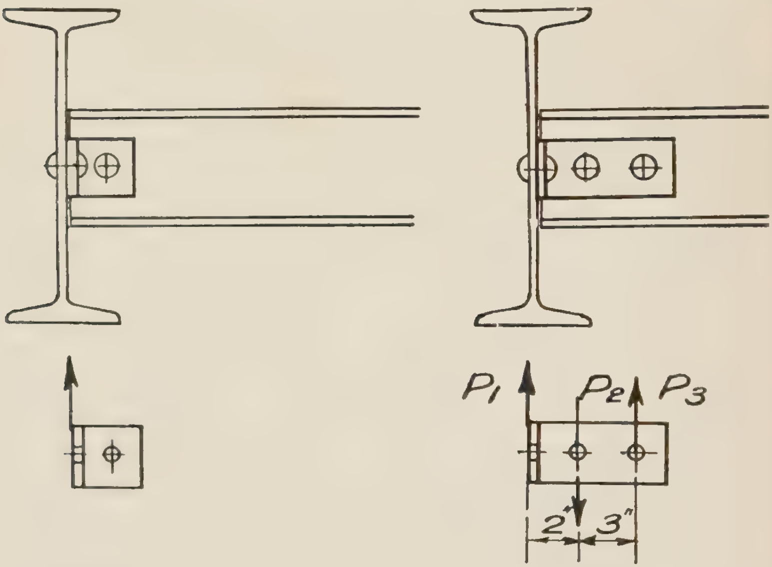
FIG. 49. Beam Connexions by Angle Cleats.

ion taking the form of two angles riveted as shown. These angles in this connexion are generally called cleats.

Where the secondary girder is of the same depth as the main girder, as may occur, the secondary has to be 'notched', as shown in Fig. 49 (*b*). This does not appreciably weaken it, as there is little bending moment at the end. From what has been said an approximate idea of the strength of such a connexion can readily be calculated, but it is somewhat complicated, particularly in the case of connexions with few rivets, by the eccentricity of the load on the connexion. It is clear, for example, that if we are dealing with very small beams, a single rivet, as shown in Fig. 50 (*a*), would be most unsatisfactory, since the load is applied to the cleat at the end, which, of course,

exerts a rotational tendency on the rivet in double shear, and this connexion would, in fact, fail in that way. It would therefore be necessary for the single cleat to have at least two rivets, as shown in Fig. 50 (*b*), to prevent this rotation.

The strength of this cleat connexion cannot be obtained by merely adding together the strengths of the two rivets, as will readily be seen by considering the forces in Fig. 50 (*b*), the forces being denoted by P_1, P_2, and P_3 respectively, and being



(*a*) FIG. 50. (*b*)

the forces exerted on the angle cleats through their respective rivets.

The force on P_1 obviously being upwards it follows that P_2 must be downwards and P_3 upwards, since if an attempt is made to reverse the direction of either P_2 or P_3 it will be found impossible to produce a system of forces which would be balanced against rotation.

Taking the distances between the forces as 2 in. and 3 in. respectively it follows that P_3 is two-thirds of P_1, and P_2 equals

$$P_1 + P_3 = P_1 + \tfrac{2}{3} P_1 = 1\tfrac{2}{3} P_1.$$

The maximum load on P_2 must be limited to that which will not overstress a $\frac{3}{4}$-in. rivet in double shear (assuming, of course,

that we are using $\frac{3}{4}$-in. rivets), which is 4·2 tons. We therefore have

$$4{\cdot}2 \text{ tons} = 1\tfrac{2}{3}\, P_1,$$

whence

$$P_1 = \frac{4{\cdot}2 \text{ tons}}{1\frac{2}{3}} = 2{\cdot}5 \text{ tons.}$$

It will therefore be seen that this cleat can only be used for reactions of 2·5 tons, although the addition of the safe shear on

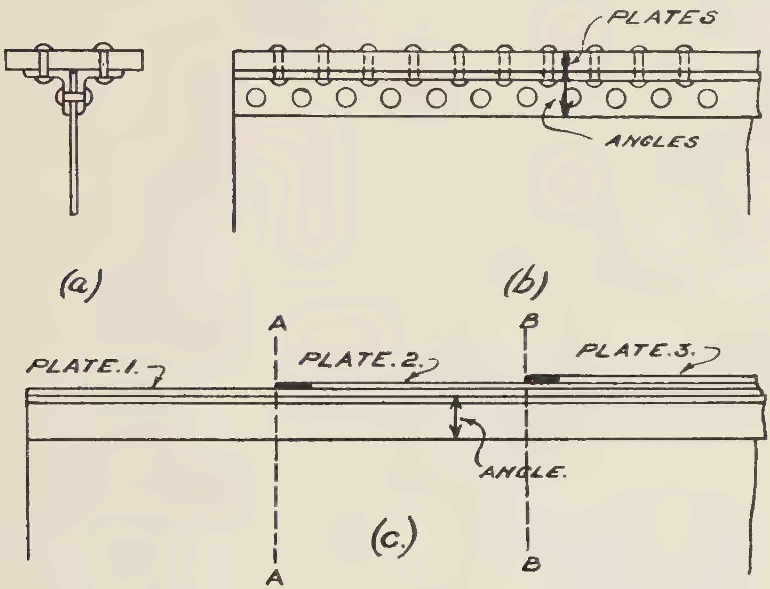


FIG. 51.

each of the rivets would have given twice 2·4 = 4·8 tons, so that in this case the eccentricity of the loading reduces the strength of the connexion to one-half, and is therefore obviously a matter requiring serious attention.

The same principle applies in the calculations of the strength of all other cleated connexions, but the effect of eccentricity is less marked as the number of rivets increases, and is less for cleats having a single row of rivets, such as Fig. 49 (*b*), than in the case of cleats having two rows of rivets, such as Fig. 50 (*b*).

An interesting point which is sometimes given wrongly in text-books in connexion with rivets connecting the flanges to the web of a girder may be stated as follows. It is frequently

stated that if the flange consists partly of angles and partly of flange plates, as in Fig. 51 (*a*), then the rivets connecting the angles to the flange plates need not be designed for the whole shear in accordance with the calculations given in the last issue, but this shear may be reduced in proportion to the ratio of the flange plates to the total area of flange.

In most practical cases this is not true. It would be true in the case of a girder, illustrated in Fig. 51 (*b*), where the flange plates run through to the end of the girder without being stopped short. But as a rule, as has already been explained, the flange plates would be stopped off in accordance with Fig. 51 (*c*), the positions for the ends of the plates being determined by the bending-moment diagram. In that case plate No. 1 and the angle may be assumed as fully stressed at the section *AA*, where plate No. 2 begins, and will still be fully stressed at the section *BB*, where plate No. 3 begins. It therefore follows that the whole of the horizontal shear between *AA* and *BB* is taken up in plate No. 2, none of the increment being taken up in the angles which are equally stressed at *AA* and *BB*; therefore the rivets connecting the angles to the plates have to be capable of resisting the whole increment of flange force between *AA* and *BB*, and no deduction should be made for the fact that the angles form a part of the sectional area of the flange.

CHAPTER IX

BASES AND GRILLAGES

THE maximum load on a stanchion occurs, of course, at the bottom, and some care is to be exercised to see that this load is adequately transferred and spread over a suitable layer of soil. The load per square foot which various soils will carry is beyond the scope of this book, but it may be stated as a guide that chalk and soft rock will carry about six tons per square foot, blue London clay about four tons per square foot, yellow London clay about two tons per square foot, and ballast two to four tons per square foot, depending on its compactness. Where the materials are soft lower pressures have to be taken, and occasionally one has to found on soils where as little as half a ton per square foot is the limit of what can safely be imposed.

Having selected the suitable carrying capacity of the soil the area of the foundation for a given stanchion load is easily determined, and the problem is then how to distribute the load over this area. One method of doing this is as shown in Fig. 52, where the stanchion is widened out by means of gusset plates, angles, and base plate to be wide enough to spread on the upper row of grillage joists. With moderate-sized stanchions carrying about 300 tons, these are generally conveniently made three in number, but for smaller grillages fewer, and for larger grillages a greater, number of girders may be required. Where three are provided the centre one may fairly be assumed to receive its load direct from the stanchion, but the two outer ones only receive load by a transmission through the gusset plates. It is therefore clear that the gusset plates in such case have to transmit two-thirds of the total load of the stanchion, and the rivets connecting the gusset plate to the stanchion must be strong enough to carry this load. This sometimes necessitates the gusset plates being 5 or 6 ft. long, which may be a great disadvantage, since if the increased width of stanchion is an objection, as it would be in an important building, it may necessitate lowering the foundations unnecessarily and increasing the cost.

This difficulty can be reduced by having at least four rows of rivets on each side between the stanchion and the gusset plates, and by using a larger diameter rivet than is the standard for the rest of the job. One-inch rivets are not uncommon in bases.

The 1909 L.C.C. Steel-Framing Act requires, in Clause 12c, that the gusset plates shall have sufficient rivets to transmit the whole of the load to the base plates, but this is an unreasonable hardship, since with reasonably good workmanship, the end of the stanchion making contact with the base plate, the load would certainly be transmitted direct to the centre grillage beam.

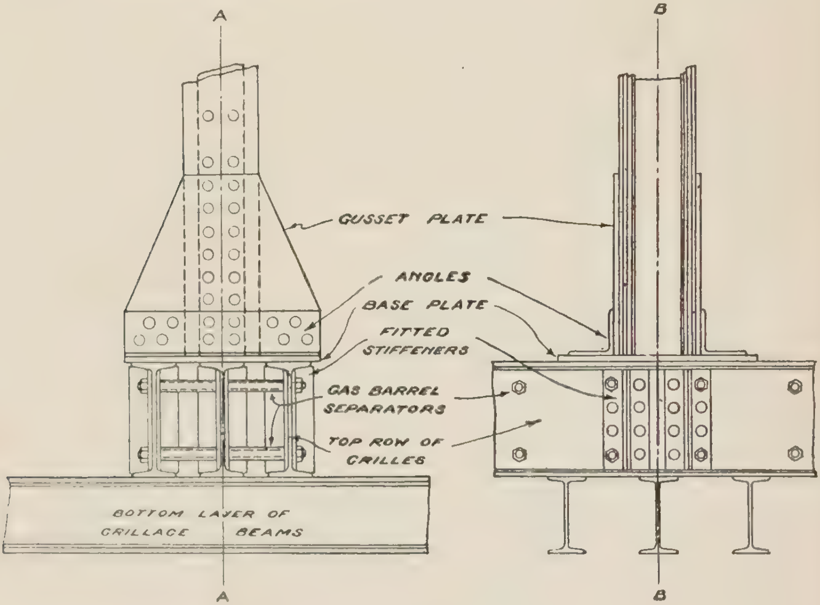


FIG. 52. Base and Grillage Detail.

The grilles are preferably kept at the right distance apart by bolts and gas-barrel separators, and should be stiffened with fitted stiffeners immediately under the gusset plates so as to receive the load from them. These stiffeners generally consist of angles back to back, and must be fitted by forging or machining to make contact with the top and bottom flanges of the grilles. The whole of the base is filled solid with concrete, both as an additional stiffness and to protect the steel from corrosion, and to get this concrete in the space between the grillage to beams should not be less than 2 in. to 3 in. The top grillage beams extend the full length of the base in one direction, the spreading in the other direction being provided for by the bottom layer of grillage beams.

The load at each intersection in the case of the bottom grillage beams is considerably less than with the top ones, and it is usually not necessary to provide stiffeners to them. The bottom grillage beams are generally four or five times as numerous as the other row and correspondingly smaller. They are evenly spaced and serve to transmit the load from the top row over the whole area of the base. The worst bending moment in the bottom grillage beams occurs on the centre line of section *AA*, and the worst moment in the upper grillage beams on the centre line of section *BB*. A much more economical base is obtained by substituting reinforcing bars instead of the bottom layer of grillage beams, as the concrete is provided in

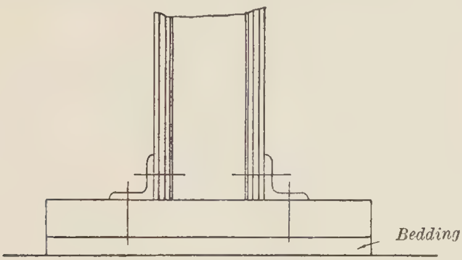


FIG. 53. Bloom Base.

either case and may just as well be called upon to assist in the work of distribution. As reinforcing rods are generally only about one-tenth of the weight of the bottom row of grillages, a further substantial saving in cost is thus effected. The same substitution is not quite so easily made in the case of the upper row of grilles when the loads are heavy, as the pressure per square foot on the concrete will be greater than is safe unless a great area of base is provided. In other words, the upper row of grillages serves to distribute the load over a sufficiently large area of concrete. This difficulty can, however, be overcome by providing a bloom base (Fig. 53) consisting of a solid slab of steel, which may be supported directly on a reinforced concrete base or raft, and obviates the necessity for gusset plates and all the objections. In very large stanchions of 600 tons or over these bloom bases may be as much as 6 ft. square and 9 in. thick, but the saving in cost in the grillages and by keeping the foundations at a higher level will frequently outweigh this disadvantage.

We will now take an example of grillage design. Let us con-

sider the case of a stanchion carrying 600 tons, which load it is desired to distribute on a soil capable of carrying four tons to the square foot.

Clearly the area of foundation required is

$$A = \frac{W}{p} = \frac{600 \text{ tons}}{4^T/\text{sq. ft.}} = 150 \text{ sq. ft.},$$

so that a base just over 12 ft. square is indicated. We will design the grill on the basis of using gusset plates as in Fig. 54. Assuming the stanchion to be 20 in. square, the forces acting on the upper row of grills would be as shown in the figure.

The worst bending moment occurs on the centre line AA. Its magnitude can be calculated by considering the forces acting on one side (either side) of this line. These forces are: a downward load of 300 tons (half the total load) acting about 10 in. from the centre line (since the load is transmitted from stanchion to the grills through the gusset plates); and an upward load of equal amount uniformly distributed over the underside of the half base, and therefore having its resultant at the mid-point of the half base, i.e. at a distance of $\frac{12 \text{ ft.}}{4} =$ 3 ft. from the centre line.

The moment about the centre line is, therefore,

$$300 \text{ tons} \times 10 \text{ in.} = 3{,}000 \text{ inch tons clockwise,}$$

and

$$300 \text{ tons} \times 36 \text{ in.} = 10{,}800 \text{ inch tons anti-clockwise,}$$

making a resultant moment of

$$10{,}800 - 3{,}000 = 7{,}800 \text{ inch tons.}$$

Adopting a safe stress of $7\frac{1}{2}$ tons/sq. inch, the section modulus required is clearly

$$z = \frac{7{,}800}{7\frac{1}{2}} = 1{,}040 \text{ inch units.}$$

Adopting the new British Standard Sections, this can suitably be provided by five $24 \times 7\frac{1}{2} \times 90$ lb. joists, having a section modulus of 203·6 inch units each, or a total modulus of 1,018 inch units. Allowing a space of $1\frac{1}{2}$ in. between joists for the insertion of concrete (this is the minimum practicable) the overall width would be $43\frac{1}{2}$ in., which determines the width of gusset plate required.

Coming now to the lower grillage, the forces are as in Fig. 54 (*b*). As before, the greatest moment will be on the centre line AA. As before, we need only consider the forces on one side

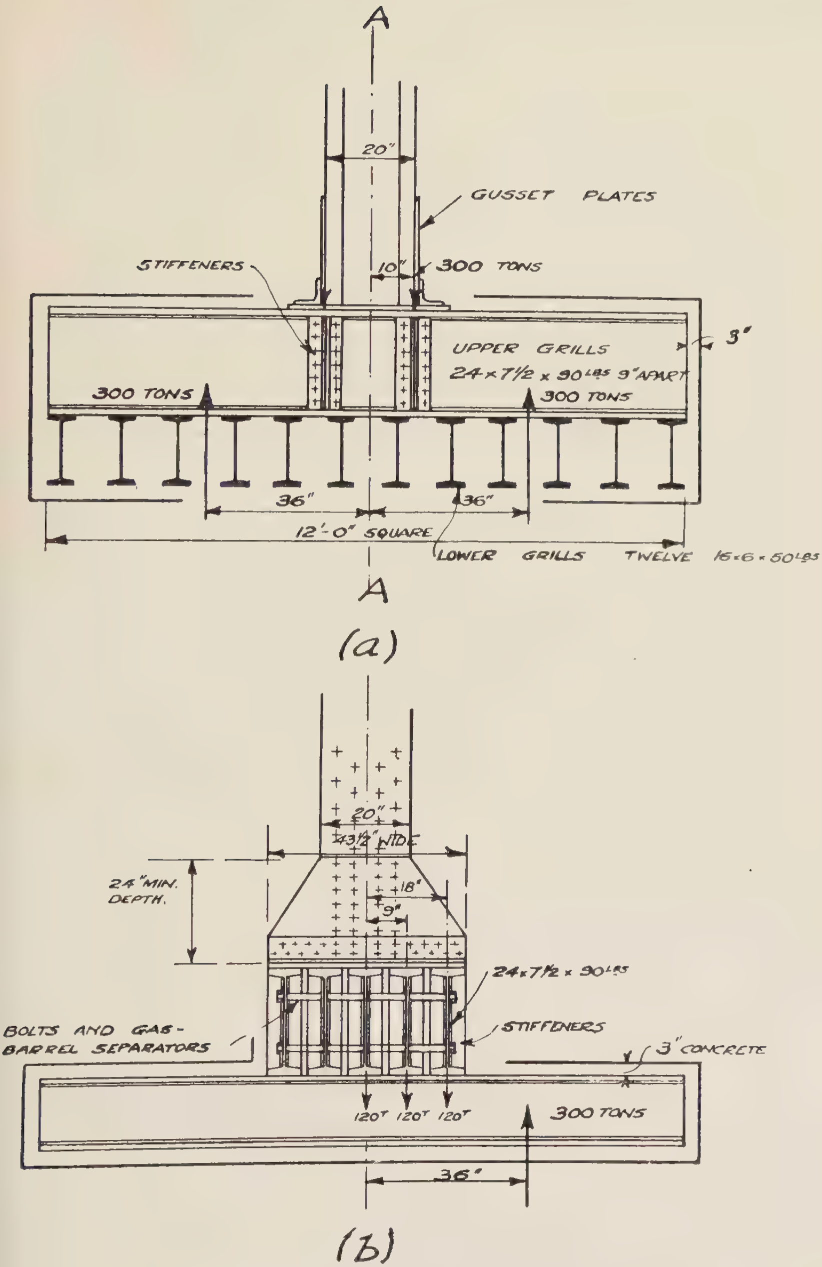


FIG. 54. Arrangement of and Forces on Grillages.

of this centre line. The downward forces on the lower grill are 120 tons $\left(\frac{600^T}{5}\right)$ from each of the five upper grills, and an upward force of 300 tons (half the total load) uniformly distributed on the half base, and therefore represented by the resultant, central on the half base, and therefore at a distance of $\frac{12 \text{ ft.}}{4} = 3$ ft. from the centre line.

The moments about the centre line are, therefore :

Clockwise. $120^T \times 9$ in. $+ 120^T \times 18$ in. $= 1{,}080 + 2{,}160$ inch tons $= 3{,}240$ inch tons.

Anti-clockwise. $300^T \times 36$ in. $= 10{,}800$ inch tons.

Resultant. $10{,}800 - 3{,}240 = 7{,}560$ inch tons.

Adopting a safe stress of $7\frac{1}{2}$ tons per square inch the section modulus required is

$$z = \frac{7{,}560}{7\frac{1}{2}} = 1{,}000 \text{ inch units.}$$

This can be provided conveniently by twelve 16 in. × 6 in. × 50 lb. joists, each having a modulus of 77·26, and therefore an aggregate modulus of 925. The concrete filling increases the strength and stiffness, even if somewhat indeterminately, which explains why an apparent deficiency in modulus may be permitted.

Coming now to the design of the gusset plates, if we followed strictly the L.C.C. regulation (General Powers Act, 1909, Clause 12*c*), which requires the gusset pieces to have sufficient rivets to transmit the *whole load* to the base plates, the gussets would need to be very large.

Even with four rows of rivets on each side (see Fig. 54 *b*), and adopting 1-in. rivets 3 in. apart vertically (the absolute minimum spacing for this size), we should only develop

$$8 \times 4{\cdot}2 = 33{\cdot}6 \text{ tons}$$

for each horizontal row of 8 rivets, and should therefore require

$$\frac{600}{33{\cdot}6} = 18 \text{ rows.}$$

These, at 3 in. apart, require a vertical length of 18 × 3 = 54 in. for the gussets. But actually it will be seen that the three centre joists of the upper grill come inside the limits of the stanchion, and would receive their load by direct bearing from the stanchion even if the gussets were omitted. If the workmanship is good, so that the end of the stanchion and the

bottom of the gusset plates are all in one plane and make contact with the base plate without any gaps, it is unnecessary in such case for the gussets to be designed to transmit more than the load from the two outer grills, which do not receive direct load from the stanchion. This reduces the number of rivets, and therefore of rows, to $\frac{2}{5}$ of what was previously required, say,

$$\tfrac{2}{5} \times 18 = 7\tfrac{1}{5},$$

say eight rows 3 in. apart, giving a minimum depth of gusset of 24 in.

Stiffeners and bolts and gas-barrel separators to the upper row of grills are required, as described on page 110. Concrete filling between the grills and at least 3 in. thick all round them (for their proper protection) is, of course, required.

The method of erecting a grill is important. If the grills are concreted up first it will be found that the stanchion, after plumbing and the fixing of inter-connecting girders above, will not touch all the upper grills (except by some lucky chance), but will probably bear at one point and require wedging at other points. The space may be too small to ensure grout filling it up solid. In that case there will be 'give', and probably unequal 'give', when the load comes on (as the building nears completion), which, besides overstraining the steel, may cause cracking of stone, partitions, &c. Even if the space is sufficient to allow of grouting solid, the pressure between the base and the top grilles is probably far higher than grout or concrete can stand without crushing.

A better method is to wedge up the whole grill, when plumbing the stanchion, maintaining metallic contact, and concreting subsequently. An alternative is to design the area of base plate to be sufficient not to overstress a concrete filling between base and grill. This generally involves the use of heavier and larger bases.

The above design may be considered typical of good standard practice. Improvements (resulting in economy of material or space) include the following:

(*a*) Shortening alternate joists in each grill, in a manner corresponding roughly to the moment diagram, remembering that the moment diminishes rapidly as we recede from the centre section.

(*b*) Substituting three plated joists for five plain ones for the upper grill. This enables the gussets (with all the depth of foundation and extra rivets they involve) to be dispensed with,

as all three joists can then receive their bearing direct. But care is necessary :

1. In not overstressing the webs in shear. Often built-up sections of plates and angles will be necessary to give adequate shear strength.

2. In making the smaller number of stiffeners adequate in area.

3. To prevent risk of lateral collapse of webs in buckling.

(*c*) Substituting reinforced concrete for the lower grill.

(*d*) Substituting reinforced concrete for both grills, and adopting a bloom or other adequate base which will not overstress the concrete in bearing pressure.

There is no space in this elementary treatment to do more than indicate these possibilities.

CHAPTER X

NEW BRITISH STANDARD SECTIONS

IT will be obvious that standard sections have to be adopted from which in practical design a selection has to be made. Steel joists and other sections are made by rolling the red-hot steel ingot through a succession of specially shaped rolls so as gradually to squeeze the metal out until after passing the last rolls it has the desired shape of the standard section. These rolls are extremely costly, and it would need a very large order indeed to justify the adoption of a section which was not quite standard; and even in such cases a delay of some months would occur while the necessary rolls were being prepared.

The choice of standard sections is a matter of great importance as some sections can be much more economical and generally suitable than others. A completely new series of British sections were adopted by the B.E.S.A. three or four years ago and are generally referred to as the New British Standard Beams. These differ from those previously in general use, chiefly in that they are somewhat lighter as a rule for a given depth. Thus, the New British Standard $24 \times 7\frac{1}{2}$ only weighs 90 lb. per ft. run instead of 100 lb. (old), and on the other hand only has a section modulus of 203·6 instead of 221·1. The consequence is that to obtain a beam of given strength it is generally necessary to adopt a section an inch or two deeper than would have been necessary with the older sections. As deeper beams are generally more economical (i.e. lighter for a given strength) than shallow ones, it follows that the adoption of the New British Standard does in general result in a somewhat more economical design than the old. This greater economy is, however, obtained at the cost of some slight increase in the depth of girders, and in the case of buildings on very valuable sites, where the maximum headroom on the various floors is important, this increased depth may outweigh the reduced cost of the steelwork. There are, however, the more numerous cases where a few inches in headroom is of less importance than a reduced weight and cost of steelwork, and on the whole there is no doubt that the New British Standards suit the general requirements of the steel trade better than the old.

The properties of the New British Standards are given in the adjoining table. In column 10 is given the safe resisting moment for a stress not to exceed $7\frac{1}{2}$ tons per sq. in. In the following column is given the safe shear on the beam stressing the web to $5\frac{1}{2}$ tons per sq. in. The adoption of this stress will in some cases entail stiffening the webs if we adopt the stress limitation in the curve of Fig. 25. Cases where such stiffening would be necessary are enclosed in brackets. It should be mentioned that the adoption of this curve is not required by the L.C.C. General Powers Act of 1909, but is strongly recommended by the author. The next column gives the safe shear on the beam if the joists are used without stiffening, but adopting the reduced stress per square inch appropriate to their particular ratio of depth to web thickness in accordance with Fig. 25.

Size. Inches.	Wt. per foot. lb.	Standard Thickness. Web.	Standard Thickness. Flange.	Area. sq. ins.	Radii of Gyration, ins. About XX	Radii of Gyration, ins. About YY	Section Moduli, ins.³ About XX	Section Moduli, ins.³ About YY	Safe Resisting Moment at 7½ T/□″ tons ins.	Safe Shear† at 5½/□″ tons.	Safe Shear* without Stiffeners. tons.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Properties of Girder Sections.											
24×7½	90	·52	·984	26·47	9·61	1·51	203·6	16·12	1527·0	(61·5)	43·3
22×7	75	·50	·834	22·06	8·72	1·36	152·4	11·73	1143·0	(54·5)	40·0
20×6½	65	·45	·820	19·12	8·01	1·31	122·6	10·02	919·5	(44·3)	32·4
18×6	55	·42	·757	16·18	7·21	1·21	93·53	7·88	701·5	(37·1)	28·1
16×6	50	·40	·726	14·71	6·48	1·24	77·26	7·49	579·5	(31·1)	25·3
15×6	45	·38	·655	13·24	6·10	1·23	65·59	6·62	491·9	(27·7)	22·7
14×5½	40	·37	·627	11·77	5·66	1·12	53·87	5·38	404·0	(25·1)	21·3
13×5	35	·35	·604	10·30	5·25	1·03	43·62	4·33	327·2	(22·0)	18·9
12×5	30	·33	·507	8·83	4·84	1·00	34·49	3·51	258·7	(19·3)	16·7
10×4½	25	·30	·505	7·35	4·08	·94	24·47	2·88	183·5	(14·3)	13·3
9×4	21	·30	·457	6·18	3·62	·82	18·03	2·07	135·2	(12·9)	12·7
8×4	18	·28	·398	5·30	3·24	·81	13·91	1·75	104·3	10·7	
7×3½	15	·25	·398	4·42	2·85	·74	10·26	1·38	76·95	8·20	
6×3	12	·23	·377	3·53	2·44	·64	7·00	·97	52·47	6·38	
5×2½	9	·20	·347	2·65	2·03	·55	4·36	·63	32·73	4·54	
4½×2	7	·19	·322	2·06	1·80	·43	2·96	·38	22·18	3·89	
4×1¾	5	·17	·239	1·47	1·58	·36	1·83	·21	13·74	3·18	
3×1½	4	·16	·249	1·18	1·19	·33	1·11	·17	8·30	2·11	
Properties of Heavy Beams and Pillars.											
18×8	80	·50	·950	23·53	7·41	1·72	143·6	17·36	1077·0	(42·7)	38·0
16×8	75	·48	·938	22·06	6·64	1·76	121·7	17·08	912·8	(35·8)	33·7
14×8	70	·46	·920	20·59	5·85	1·80	100·8	16·67	756·0	(29·4)	29·3
12×8	65	·43	·904	19·12	5·05	1·85	81·30	16·30	609·8	22·8	
10×8	55	·40	·783	16·18	4·22	1·84	57·74	13·69	433·1	17·3	
10×6	40	·36	·709	11·77	4·17	1·36	40·96	7·25	307·2	16·2	
9×7	50	·40	·825	14·71	3·76	1·65	46·25	11·48	346·9	15·1	
8×6	35	·35	·648	10·30	3·34	1·38	28·76	6·51	215·7	12·1	
6×5	25	·33	·561	7·35	2·48	1·16	15·05	3·95	112·9	8·20	
5×4½	20	·29	·513	5·88	2·06	1·06	10·01	2·93	75·08	5·83	
4×3	10	·24	·347	2·94	1·63	·67	3·89	·88	29·20	4·09	

* Adapting shear stress in Fig. 25.

† Stiffened when required. Joists requiring stiffening are in brackets.

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